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A STUDY OF 1940 GRADE IX EXAMINATION ANSWER PAPERS IN ALGEBRA AND GEOMETRY

A DISSERTATION
SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR
THE DEGREE OF MASTER OF ARTS

COLLEGE OF EDUCATION

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CHAPTER I

INTRODUCTION

" .... a man's reach should exceed his grasp,
Or what's a heaven for ?"

Browning

This thought, which Browning attributes to Andrea del Sarto, the Italian Renaissance painter, expresses an ideal, held in the minds of many people -- but often inarticulate. Physical scientists, never content with progress to date, are constantly endeavouring to perfect new processes and new instruments -- be they simple or complex, large or small -- to make man's control of his environment more complete. Medical scientists and research workers, urged on by new problems and baffling old ones, are at war incessantly with death and disease, in order that man's life may be longer and free from pain. In like manner, philosophers have long toiled to unravel the mystery of the "what" and the "why" of life, with the end in view, that man's existence be given greater meaning and reason. In a small measure, teachers might be considered as philosophers concerned in their own way with one aspect of the philosopher's problem. They are charged with carrying out some phases of the preparation of youth for the multiplicity of problems that will face them when they become adults.

As is the case with almost every phase of human endeavour, their task is rather well defined -- but the methods and the materials to be used in the pursuing of that task are subject to constant questionings, evaluations, modifications, and substitutions. Each school subject has been examined to determine its worth to hold a place in the

training of our youth. Methods, as well as subjects, have been exhaustively studied in order that their position in the composite pattern of which they form a part might be evaluated. Among the methods considered might be mentioned the methods of imparting instruction, and of evaluating the measure of success of that instruction.

Examinations have long been a factor of moment in any system of education. Discussions have waxed at length and in great earnestness as to their very validity. Further discussions have been held and investigations carried on as to the best type to be used. Lastly, investigations have been made as to what could be learned from the examinations themselves. Such is the nature of this investigation.

The actual marks on an examination paper are of particular consequence to the person who wrote it. However, to a teacher, concerned as he is with the teaching of other students, it is the mistakes which furnish the material for thought. What are the common mistakes? What percentage of students make these mistakes? How can the teaching be adapted so that the on-coming school children can be guided to avoid these mistakes and take with them clear-cut concepts that educationists believe will be of value, in both the utilitarian and aesthetic senses, to them in the future?

The results of this investigation are a contribution to the consideration of a small aspect of a great and complex problem. It is presented with the earnest hope that it will do something to throw some light on one question confronting education in the Province of Alberta.

CHAPTER II

PROCEDURES FOLLOWED IN CARRYING OUT THE STUDY

Materials and Methods of Sampling

Since this investigation was carried out under the direction of the College of Education, University of Alberta, that institution undertook the necessary steps in securing the material for the study. They obtained the approval of the Department of Education of the Provincial Government for the project, and also made provision for securing the papers.

It is assumed that the officials of the College of Education suggested the principles under which the sampling was done. It was decided that the sample should be large enough that the results would be conclusive. To that end, samples of Algebra and Geometry examination answer papers totalling four hundred and fifty papers in each subject were obtained. Further, it was deemed advisable that the results from urban school centres with large student populations, from town schools with smaller populations, and from rural schools with limited numbers of students be obtained, in order that no one type of student should contribute all the material. So it was that the four hundred and fifty papers were selected, a third of the total from each of city, town, and rural schools. This still left samples large enough (one hundred and fifty of each) that the conclusions derived from their study could be considered or proved valid.

The method of sampling, after the aforementioned principles had been laid down, was purely mechanical. Within the limits of the available material, every twentieth paper was selected. This insured that no one school, or relatively small group of schools, supplied all the papers

for any one sample. Thus it was that out of the whole group numbering upwards of ten thousand, the sample comprised four hundred and fifty papers, or, roughly, about one-twentieth of all the papers written in the subjects under consideration.

Questions Raised

Once the papers were at hand, two major problems presented themselves. What was to be attempted? How was this to be done?

Many questions have undoubtedly beset every teacher of Grade IX Mathematics in Alberta, relative to his work in this subject. These questions have been of two kinds, those relating to the "strategy" of the subject, and those relating to "tactics". Under the heading -- "Strategy" -- come questions as to the validity of the course -- its usefulness in life, its value as a preparation for the professions, and other broad questions in the realm of educational philosophy. In regard to "Tactics" -- the realm of educational psychology -- the questions were of a different nature. What standards are obtained by students throughout the Province with regard to the paper as a whole and to certain questions in particular? How do the students compare who have been prepared in the city, town, and rural schools? What types of mistakes are common to most students? Are the mistakes all due to mathematical concepts introduced at the Grade IX level only? Are mechanical errors very prevalent? What remedial work might be done to anticipate and remove the common defects in students' work? Are the mathematics tests (to name only our immediate concern) doing the work for which they were designed? It is in this field of "tactics" -- educational psychology -- that these and many other questions are raised, which beset all earnest

teachers. An attempt will be made in the progress of this study to indicate answers to them.

Outline of Method of Working

In regard to the second question raised, namely, "How were the conclusions to be arrived at?", three investigations were carried out. The first was to list the types of thinking; the second was to tabulate the types of answers, and the third to treat the scores statistically.

As a first step in doing the actual work with the papers, a paper, carefully worked out by the investigator, was analysed. Every question, or part of a question, was considered individually, and a list made of all types of work that went into the answering of the question, from reading the question to transferring the answer from the working space to the answer space. This was made as comprehensive as possible, for mistakes were known (by a cursory examination of students' answer papers) to arise for every type of thinking and work required. Further, it was noted that many of these concepts or abilities were introduced prior to the Grade IX level.

After all questions had been examined for the types of thinking required in their solution, a check chart was drawn up for each question. This made provision for keeping city, town, and rural results separate. Also spaces were provided to note the number of correct or partially correct answers, the number not attempted, the number of probable or likely errors, and the number of exceptional errors. Then each paper was examined in turn for the results, on that paper, of the particular question under consideration, and the result was tabulated in the appropriate space. After the results from the four hundred and

fifty papers had been secured, the results were summarized for the whole sample, and for each sub-population in that sample. This procedure was repeated for each question. In this way, a reasonably comprehensive idea of the work done in each question was obtained as well as of the relative results in city, town, and country papers. This formed the basis for criticism of the answers and for suggested remedial work.

Lastly, the actual scores, were considered. The arithmetic means were worked out in each case; whole sample, city, town, and rural samples, for each of the Algebra and Geometry papers. The median scores were calculated for each of the eight groups of samples. The standard deviations of each of the groups were found. Rank correlation coefficients of Algebra and Geometry scores within the sub-groups were calculated. Graphical representation of the scores was made.

It would be presumptuous in a work of this nature, for one person to lay down a group of suggestions that supposedly contained the remedial work of a subject. Such is not the intention. However, from a study of answer papers that are not readily available to all that would like to examine them, some trends in remedial work are indicated, as well as specific suggestions in regard to certain wide-spread errors.

CHAPTER III

ANALYSIS OF ALGEBRA ANSWER PAPERS

Method of Presenting the Findings

In the following pages, the actual results of examination of students' answers are given. For each question, there will be found the major types of thinking required, the results obtained by the pupils, and a note of the predominant types of errors together with the frequency with which they occurred. Following these is an analysis of the findings, and suggested ways and means of remedying the faults.

When it was thought that constructive criticism might be derived from their study, the various types of mistakes in candidates' answers were recorded separately. Because other mistakes gave no clue as to how they were arrived at, and also because of their great diversity, such mistakes were grouped as miscellaneous. It was thought that no good purpose would be served by considering them separately.

Examination of Individual Algebra Questions

Values		
	1. If (-3x) and (-10xy) are two numbers	
1	(a) their sum is	
1	(b) the remainder when the second is subtracted from the first is	
1	(c) their product is	
1	(d) the sum of the squares of the two numbers is	

1. (a) The outstanding fact to be recognized in Part (a) of the question is that the terms are dissimilar, and, as such, cannot be combined into a single term. The Rule of Signs, the use of brackets, and the double use of the minus sign, i.e., as a sign of quality, and as a sign of operation, have to be applied.

Results:	City	Town	Rural	Totals
Correct	23	21	20	64
Incorrect	127	128	130	385
Not attempted	0	1	0	1

In regard to the wrong answers, the answers "+13xy", "-13xy", and "$-13x^2y$" were very numerous. Well over 67% of all students gave the three above-mentioned answers. The other mistakes were not tabulated owing to the great diversity of answers, with no apparent reason for their use.

1. (b) This example requires knowledge of dissimilar terms, together

with the concept of subtraction. Subtraction, in turn, involves further use of the Rule of Signs.

Results:	City	Town	Rural	Totals
Correct	13	14	16	43
Incorrect	132	130	133	395
Not attempted	5	6	1	12

The most common incorrect answer was "+7xy". Over 50% of all students gave "+7xy" or "-7xy". Another appreciable group gave as their answer "+7y" or "-7y".

1. (c) This is again combining dissimilar terms, but by means of multiplication. In this type of work, a single term results. Skills involved in this question are: Rule of Signs, the Index or Power Rule, and expressing a product without the multiplication sign.

Results:	City	Town	Rural	Totals
Correct	98	116	110	324
Incorrect	46	33	38	117
Not attempted	6	1	2	9

The incorrect answer "$-30x^2y$" was the most common in each of the various groups of papers. Other common ones were "+30xy" and "-30xy" -- indicating trouble with signs and powers. Miscellaneous errors accounted for from 10% to 16% of the incorrect answers.

1. (d) The principal skill required in this question is the squaring of algebraic expressions. The Rule of Signs and the Index Laws must be used. Of course, after this is done, the two dissimilar terms have

to be added, or, at least, addition must be indicated.

Results:	City	Town	Rural	Totals
Correct	10	14	7	31
Incorrect	119	128	128	375
Not attempted	21	8	15	44

Approximately 33 1/3% of students in the city, and over 50% of students in each of the town and rural areas made the mistake, after squaring the numerical coefficients correctly, of combining the dissimilar terms into a single term with a variety of literal factors. In all cases, about 33 1/3% of students gave various incorrect answers not covered by the above criticism. The answers were of so diverse a nature that nothing was to be gained by tabulating the individual errors.

Running throughout three of the four parts of this question, (a),(b), and (d), is definite evidence of a complete lack of ability to handle dissimilar terms. This is further borne out by results from Part (c), in which dissimilar terms may be combined into an answer of a single term. It will be noted that the percentage of correct answers in Part (c) is reasonable in comparison with the other three sections of the question, where the percentage of correct answers is very low.

One feature of these results should be very thought-provoking. Only a very few students admitted that they were unable to do the question -- those people did not attempt it. In all other cases students put down an answer -- and they must have believed it to be correct. The great number of incorrect answers indicates that a very large percentage of these students just didn't get the concept of dissimilar terms, and

knowledge of what could and could not be done with them. The question raised is not that they were wrong in such great numbers, but that they thought they were right.

Other difficulties encountered in the question revolved around the ideas of the Rule of Signs, in the fields of multiplication, addition, and subtraction, and the Index Law.

Values	
	2. In the expression $4m^3 + 6m^2n + 8mn$
1	(a) the second term is
1	(b) the factors of the third term are
1	(c) the greatest common factor of all the terms is

2. (a) The only concept necessary to answer this question correctly is contained in the meaning of the word "term".

Results:	City	Town	Rural	Totals
Correct	110	95	91	296
Incorrect	26	39	47	112
Not attempted	14	16	12	42

The types of mistakes were not tabulated, owing to the great variety of incorrect answers.

Among those students who didn't answer the question correctly or who didn't attempt it -- amounting as they do to upwards of 33 1/3% -- surely there are some who understand the idea behind the question, but who are not able to interpret the question in the light of their experience. This concept is introduced very early in the course, but it must have been considered too easy for teachers to dwell on it. As has been noted before, nothing should be assumed to be too easy for students beginning to work in a new medium.

2. (b) Here, a correct answer depended upon ability to recognize what

was meant by the "third term" and by the word "factors". Pupils had to have knowledge of how to write and punctuate items of a series.

Results:	City	Town	Rural	Totals
Correct	73	81	60	214
Incorrect	63	57	76	196
Not attempted	14	12	14	40

Among the incorrect answers, about 16% of all pupils gave the literal factor only, while a substantial group apparently knew what was wanted, but didn't get credit because they failed to separate the factors properly in writing them. Answers in this category were: 8 mn and 8 m n.

Other than to note that a comparatively few people -- under 50% of all pupils -- had this question correct, the only observation is that too little care is taken in expressing an idea -- that conventional English rules must be followed in putting mathematical answers on paper. This question of pupils expressing themselves according to well-established, if not well-formulated, rules of English is linked with their inability to interpret the printed word adequately. Either through indifference or lack of recognition of the difficulty and seriousness of the situation, the reading of instructions and the writing of answers in good English is not done.

2. (c) The candidate had to recognize that a term was wanted -- the largest term that would divide evenly into each of the three terms. This concept, used in Arithmetic, is introduced with, and forms an integral part of, the work with fractions about the Grade VI level. The use of

literal factors is a Grade IX concept.

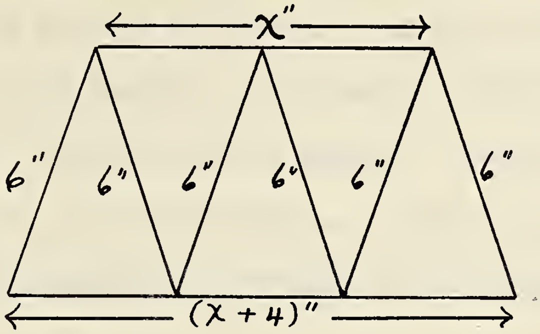
Results:	City	Town	Rural	Totals
Correct	2	2	3	7
Incorrect	136	128	128	392
Not attempted	12	20	19	51

The two most common errors were to give only the highest common factor of the literal factors, i.e., "m", or the highest common factor of the numerical factors, i.e., "2". Other than these two mistakes, over 66 2/3% of all students attempting this question had answers other than the correct one, or the two above-mentioned mistakes.

To a mature student of Algebra, this question would appear very elementary. Evidently the concept, or chain of concepts used in finding the greatest common measure, with the emphasis at present given to it, is beyond all but the very brightest of Grade IX students.

As previously stated, in connection with the criticisms of individual parts, above, it appears evident that the work in factoring, and definitions, such as "term", are taken too lightly. If concepts used in factoring and nomenclature used in the study of a subject are important enough to learn, they should be learned to the point where they are meaningful and can be put into practice. If the time at present spent gives results such as are indicated above, it is wasted. Either greater emphasis should be placed on it, or it should be considered worthy of nothing more than the casual treatment it now receives, and as such, should not be used as examination material.

Value
3 3.



A wire frame has the shape and dimensions shown in the diagram. Give in inches, without using X in the answer, the total length of wire in the frame.

3. The first step in solving this problem is to arrive at the relationship that x = 8. This could be done by inspection, or by solving the equation: $x = 2/3(x + 4)$. If the equation is solved, all mechanical aids to solving an equation must be applied: clearing of fractions, transposing, use of brackets, etc. When the value of x is attained, the perimeter is found by adding the lengths of all the parts of the wire frame.

The question states that x must not be used in the answer, which should mean that enough data are given to enable the student to solve for x. Further, the desired unit is indicated, although leeway was given in the marking to allow for answers in units other than that specified.

Results:	City	Town	Rural	Totals
Correct	50	45	44	139
Incorrect	66	81	88	235
Not attempted	34	24	18	76

Wrong answers for this question were very varied. Some of the common ones were: "36","80","72","60","16","32", and "40". The reader may conjecture what methods were used to arrive at such results -- nothing was shown on the papers. In a small percentage of cases, x or some other unknown, was used in the answer. The miscellaneous answers were, in many cases, beyond all reason. Among this latter group might be noted these: "4","5660","23,376" and, of all things "16 square inches".

These impossible answers indicate that many students treat such a question as this as wholly unreal. No estimate based upon reason was used to test their answers. If it had been, such answers wouldn't have been offered. Testing, on the basis of common sense, needs to be stressed.

In too many cases, there was no work to show how the answer was arrived at. If the answer is correct, this does not work a hardship on the student, but, if the answer is incorrect, it does, in some cases, become a handicap. Since there was no space provided near the question, it was presumed that the student would use the blank side of the sheet. That was not done. Probably foolscap was used by some students although the instructions were to the contrary. Partial credits are awarded for partially correct work, but only on the presentation of evidence of that correct work.

Lack of evidence as to how the incorrect answers were arrived at, makes constructive criticism of this question almost impossible.

Values

1 4. (a) If $6x^5$ is divided by $-9x^5$ the quotient is

2 (b) In a certain question the divisor is X, the quotient is XY and the remainder is 5. Find the dividend.

3 (c) $\frac{n + \ldots}{12} = 1\ 3/4$. If n = 10 what number should be written in the blank?

4. (a) In order to find the answer to this question, pupils must realize what "quotient" means. They must be prepared to handle signed numbers, must understand the concepts inherent in indices, must be able to cancel numerical and literal factors, and must be able to conceive of a fractional answer.

Results:	City	Town	Rural	Totals
Correct	28	41	43	112
Incorrect	92	84	88	264
Not attempted	30	25	19	74

The highest percentage of errors resulted because of misapplication of the rule of indices, viz., almost 33 1/3% of all answers contained a power of x. The sign was wrong in about 10% of the cases. Another 10% had a coefficient of "1 1/2", indicating improper interpretation of the question, or the terms in which the question was couched. A fairly large percentage of pupils did not attempt the question.

The results indicate, that the concepts relating to division of

algebraic expressions were not mastered. No doubt, the individual ideas concerned -- signs, cancellation, etc. -- could be handled by many students who gave incorrect answers. The chain of operations, all of which had to be correct, was too difficult for most. This is a very serious matter, as skill in these operations forms the basis of much subsequent work, in the work of division of polynomials, factoring, and equation and problem solving.

4. (b) No other type of question that could be formulated tests all the concepts of division, in its formal sense, better than the example under consideration. It includes a knowledge of the terms used in the formal process of division, together with the relationships that exist between the various quantities. A knowledge of indices is essential in order that the dividend may be written in the most concise form.

Results:	City	Town	Rural	Totals
Correct	49	59	49	157
Incorrect	66	66	79	211
Not attempted	35	25	22	82

The most common mistake was the answer: "x^2+xy+5". This answer shows that the various concepts required by the question were known, but that the relationships of the various quantities were not properly understood. In some cases, evidently, the correct answer was found, but it was improperly put down, as "x^2xy+5". It would seem that the student intended the second "x" to indicate multiplication, but the examiner could not make this assumption, and had to give the answer no credit. There was a wide range of miscellaneous incorrect answers.

Questions of this character, as well as many others of a similar type, should be treated very carefully in a form that shows the relationship of the component parts. There was no evidence that this was done.

Again such an exercise as this is very easily checked. This was not done. Pupils should be led to check material as a matter of course -- it should be a universal habit. The facts of division may become hazy or be forgotten, but habits of doing careful work and checking answers is a habit of lasting value -- probably the one justification for subjecting the entire school population to exercises in division of monomials.

4. (c) This equation is the common linear equation in one unknown. The way in which it is stated cloaks this fact. Hence, the first step in its solution is the substitution of 10 for the "n", and placing some symbol in the blank space. After this has been done, the mechanical operations, of clearing of fractions and transposing of terms, must be carried out.

Results:	City	Town	Rural	Totals
Correct	90	91	88	269
Incorrect	42	45	53	140
Not attempted	18	14	9	41

Incorrect answers varied from "84" and "68 4/7", down to "11/12" and similar quantities. There was very little evidence as to how the question was worked -- some answers may have been worked out mentally or the work done on paper other than the examination booklet. There was no way in which the wrong answers could be analysed to find the nature

of the mistake, upon which to base constructive criticism.

The prevalence of answers in the extremities of the range noted above indicates that the work was done in some formal unreasoning manner. A simple check for an approximate value would forestall the use of such answers. A careful check of the root in the equation would have indicated an error. This was left undone in most cases.

Value

4 5. If U is 10; g is 32; and t is 20, find S if $S = Ut + 1/2\ gt^2$

5. This question presents an example of straight forward substitution, with the exception of the last term in which a power of "t" occurs. One convention that must be known is that the power is applied to the one factor only. The proper solution demands care in the substitution and correct mechanical work.

Results:	City	Town	Rural	Totals
Correct	56	61	54	171
Incorrect	92	83	95	270
Not attempted	2	6	1	9

The common errors were: doubling the 20 factor, rather than squaring it; and squaring the product 32 x 10, (the 2 having been cancelled out of the 1/2 and the 20). An appreciable percentage of the pupils actually put the values in the wrong places, i.e., they used "10" in place of "32" etc., and then went on to carry the work through correctly. About 10% of each of city, town, and rural pupils made mechanical errors after the substitution had been done correctly.

In the majority of cases, there was no systematic attempt made to put the equation down, to record the given values, to substitute, to cancel, and to multiply and add. One might be inclined to say that such a set of formal, tedious steps is not necessary -- but the evidence points to the fact that a question cannot be done as successfully with-

out the formal steps. Too many errors creep in when the work is attempted haphazardly -- some parts put down, and some done mentally. Students suffer if they do not show their work, since partial credits are only awarded if some correct work is shown.

From the standpoint of the Algebra concerned, the major difficulty was the use of the square of "t". Since much of the use of Algebra in scientific fields and higher mathematics is concerned with this very important type of work, substituting in formulae, more experience in its use should be provided for students. A topic such as indices finds its widest application in formulae, and as such deserves emphasis. No doubt should be allowed to remain in the students' minds about the conventions and rules of the use of indices.

Values

1 6. (a) The base of a triangle is a inches and the height is b inches. The area is ☐ sq.ft.

2 (b) A cylinder of height h inches has a base radius of r inches. The formula for the volume of the cylinder in cubic inches is ☐

3 (c) If C and F are temperature readings on the Centigrade and Fahrenheit thermometer scales, then $F = 9/5\ C + 32$

It follows that C (in terms of F) = ☐

6. (a) At first glance, which was all that a good many pupils gave this question, it appeared that all it required was that the formula, $A = 1/2\ ba$ be given. However a change was needed -- the dimensions were given in inches, and the answer was required in square feet. The fact that there are 144 square inches in a square foot, is first introduced at the Grade V level.

Results:	City	Town	Rural	Totals
Correct	24	21	23	68
Incorrect	115	127	126	368
Not attempted	11	2	1	14

In city, town, and rural areas, slightly over 33 1/3% of the students had the correct answer except for the unit. Others had difficulty because they didn't know how many square inches there are in a square foot, or because they couldn't see whether the number of square inches was to be diminished or increased to give square feet. This

group had answers which included the following: "ab/12","ab/24","72ab", and "6ab". Still others added "a" and "b", or squared one or the other before adding or multiplying. There were other miscellaneous errors other than the ones specifically mentioned.

A mature student of mathematics would say that there was no particular difficulty in this question. This is not a true picture from the standpoint of the student finishing Grade IX. Over 80% had the question incorrect. Possibly the greater portion of these mistakes can be attributed to plain carelessness in reading. Candidates didn't bother to find out what was asked.

Other mistakes were due to apparent inability to change from square inches to square feet -- probably because of the literal value of the area in the example. Some pupils multiplied by 12, and some by 144; others divided by 12. Since this concept of changing from one unit to another in denominate numbers was first introduced in Grade V, two reasons might be given for the failure of these students to handle the situation correctly. Either it was assumed by the Grade IX teachers that pupils understood the principle, and were thus not drilled in it, or the putting of a familiar concept in a novel setting (using literal dimensions) created a new problem which rendered former concepts meaningless. More likely the latter is the truer picture.

Teachers should not assume that operations familiar in arithmetical situations, present no more difficulty in a situation dependent on algebraic concepts. They must plan their work on the assumption that to embody old concepts in the new medium requires extra effort, drill, and much experience.

Nothing can be done with that pupil who is too careless and

indifferent to read and use the knowledge and power he has.

6. (b) To answer this question, a pupil must know that a cylinder is a prism with a circular end, or, which amounts to the same thing, that volume can be found by multiplying the cross section area by the length or height. It becomes necessary to use literal values in the process of finding area -- substitute πr^2 for the area of the base, and use h for the height. Since the units are all in inches, the answer is in cubic inches. It is necessary to know that when a formula is asked for, two sides of an equation must be given, i.e., $V = \pi r^2 h$.

Results:	City	Town	Rural	Totals
Correct	68	66	76	210
Incorrect	61	69	64	194
Not attempted	21	15	10	46

In the group of correct answers, 50% gave the formula required, whereas the other 50% gave the right member only. Up to 16 2/3% gave answers consisting of a coefficient with "hr" as the literal factors. Some added "h" and "r". The other errors were widely distributed as to type.

Even though this question is more difficult than the one immediately preceding it, the results are markedly better. This would indicate that the question of units is one which is not properly understood by our students -- even those leaving Grade IX.

Lack of drill is thought to be the one main cause of mistakes in this question. Pupils can learn the facts required to find the volume of a cylinder if given sufficient experience in doing it. As has

been found to be the case before, the placing of a familiar concept in an unfamiliar setting creates a new problem. It cannot be stressed too greatly that teachers must become aware of this new difficulty, and govern themselves accordingly.

6. (c) This question involves manipulation of an equation -- given two unknowns, to express one in terms of the other. A proper understanding of what is required is necessary -- C is to be the left member, and all else, in its proper relationship, is to become the new second member of the equation. Use of brackets to indicate two or more terms considered as a single factor, or some other comparable device, is essential. A thorough understanding of the scientific concepts in connection with thermometer scales is an asset here.

Results:	City	Town	Rural	Totals
Correct	21	15	13	49
Incorrect	99	98	109	306
Not attempted	30	37	28	95

Slightly more than 20% of all students transposed the "32" correctly, and divided by "9/5", but failed to include the "F-32" in brackets. These pupils received partial credit for their work. Other errors noted were: pupils didn't divide by 9/5 correctly; other students didn't change the sign on transposing the "32"; others gave an answer in numbers with no variables present; still others purported to find "F" (although it was already given), or had answers including both "C" and "F".

This example is one which occurs specifically in the Grade IX

Science Course. The results are not indicative of any mastery of the principle either from the standpoint of Algebra or General Science. Pupils just don't try to reason such a question out, nor are they in command of the mechanical operations to the degree that they are able to say they are correct whether they understand it or not. More emphasis on both aspects is necessary if mastery is to be achieved.

The mechanical operations demanded in the question are fundamental -- clearing of fractions or dividing by a fraction, and transposing. Complete comprehension of the subject matter is not essential in order that these things be done correctly. In this regard, no doubt many who would profess to be able to do these operations successfully, made errors when confronted with the two unknowns.

In regard to a complete comprehension of this question from the scientific viewpoint, pupils could easily get a check on the accuracy of their Algebra by choosing two standard points, e.g., 0 degrees Centigrade and 32 degrees Fahrenheit, and checking their expression with the two constants. This was not done on any of the papers examined.

The mistakes in the mechanical operation of dividing by fractions can not properly be laid at the door of Grade IX Algebra.

Values

3 7. When $2n^5 + n^4 - 12n^3 + 7n^2 - 19n + 15$ is divided by $2n^2 + 5n - 4$ the quotient is

1 and the remainder is

2 Check by numerical substitution using $n = 2$

7. This question is one of the few on this paper which must be done by means of a memorized process -- it is of a very mechanical nature. The elements used in the whole process are: Rule of Signs, the Index Laws, and multiplication and subtraction of algebraic expressions. The determining of the trial divisor is very important -- involving as it does all the concepts above mentioned, except subtraction.

Results:		City	Town	Rural	Totals
(a)	Correct	48	61	67	176
	Incorrect	87	70	65	222
	Not attempted	15	19	18	52
(b)	Correct	42	55	60	157
	Incorrect	90	76	72	238
	Not attempted	18	19	18	55
(c)	Correct	11	5	11	27
	Incorrect	58	51	50	159
	Not attempted	81	94	89	264

In tabulating the results for Part (a), the work was examined and the nature of the first error was noted. No further examination of the work was attempted when a mistake was found. The most common error was a mistake in the Rule of Signs with regard to subtraction. Another difficulty was in finding the trial divisor -- dividing a monomial could not be done by some students. Mistakes in mechanical operations were fairly common -- ranging up to nearly 10% of all the errors. A very few people copied the question incorrectly into the blank space where it was to be worked, and then proceeded to do the division correctly.

Naturally one would believe that a correct remainder depended on correct work to the end of the question; that is, if the division were done correctly, the remainder would be correct. There arose, however, the anomalous situation, that while there was actually an error in the last step of the division, the quotient was correct. In this case, the pupil, and it is believed unjustly, received credit for the division but lost the credit for the remainder. In some other cases "-3" was given as the answer. Only a very few put the number "3" over the divisor, as the remainder.

In regard to the check, only about 14 2/7% of those who attempted it had it correct -- and well over 50% didn't even attempt it. This is partly explained by the fact that in some cases the pupil did not complete the division, and had nothing to check.

The results show that only the top 40% of these Grade IX students had mastered division of polynomials. Perhaps this is enough -- this type of work might be just the type of work to be the instrument for segregating those that should go on to the academic high schools

from those that should be directed into other endeavours. If this is so, then the question in point served its purpose well.

If the example is considered a test in carrying a large number of comparatively easy steps correctly to completion, it indicates that our Grade IX students don't measure up to the task. Too often there was evidence that the pupils had some idea as to how to proceed, but, the multiplicity of detail and consistent accuracy required proved too much for them, and they gave up the struggle. This was particularly true of the check.

The lack of a systematic, precise form of doing the question and a disregard for the help to be obtained from a neat copy of work were noted. Some of the scrawls put down in the name of Algebra defeated the purpose for which the work was intended. They were quite unintelligible even to one who knew in advance what to look for.

Values	
1	8. (a) If $1/8\ x = 1/8$, then x is
1	(b) The numbers x and 160 differ by 25 x is either or
2	(c) If $4n - 0.6 = 1000$, then n is
1	(d) $(-b^5) \times (-b^2) =$
1	(e) $(-5d)^3 =$
1	(f) $-(4st)^2 =$
2	(g) $(-10x^3y + 6x^2y^2 - 2xy^2) \div (-2xy) =$

8. (a) This is really a single step equation, to be solved by clearing of fractions, by dividing by 1/8, or, simply, by inspection. If it is thought of at all, this answer should be obvious to any person, regardless of specific training in Algebra.

Results:	City	Town	Rural	Totals
Correct	113	111	109	333
Incorrect	32	37	39	108
Not attempted	5	2	2	9

About 10% of the students answered that "$x = 0$". Another 10% said that x was either "1/8" or "8". Other answers varied from "1/64" to "8x" and "8/x".

Mistakes that resulted from attempting this question would appear to be due to the following: using a snap judgment, and failing to check; using a rule wrongly and failing to check; or failure to use ordinary common sense. It must have appeared too easy to require spending any time considering it -- and therein lies much difficulty here and in other questions.

8. (b) The unknowns must be thought of as being two numbers -- one greater and one less than 160. If the results were arrived at without the use of pencil and paper or the use of algebraic equations, addition and subtraction are all that are necessary. The representation of this question by equations, requires that the operation of transposing be done correctly.

Results:		City	Town	Rural	Totals
(a)	Correct	123	119	116	358
	Incorrect	10	15	21	46
	Not attempted	17	16	13	46
(b)	Correct	135	120	130	385
	Incorrect	8	20	11	39
	Not attempted	7	10	9	26

For purposes of tabulating results, the Part (a) answer was considered to be the answer "185" and the Part (b) answer was "135". The committee marking this paper, by agreement with the General Chairman, awarded one mark to the second answer to facilitate the marking.

The results show that this question was very well done. One observation is presented for what it is worth -- the answer arrived at

by subtraction was given by a few more students than the one requiring addition.

8. (c) To solve this equation, a pupil would have to be familiar with the operations of transposing terms, and clearing of the coefficient of the unknown. This involves addition and division of decimals.

Results:	City	Town	Rural	Totals
Correct	90	70	83	243
Incorrect	52	68	61	181
Not attempted	8	12	6	26

The largest percentage of errors, about 15% of all answers, occurred in putting the decimal in the wrong place. Mechanical errors in addition or division, apart from the decimal fractions involved, accounted for about the same percentage of errors. This is Grade VII Arithmetic. From the standpoint of Algebra, some 7% subtracted the fraction from the whole number before dividing.

The results here are quite discouraging. There is a decided weakness in mechanical ability and in the concepts of decimal fractions. This is to be deplored because any further work in mathematics depends upon near perfection in these two fundamental concepts. It may be a case of a concept being temporarily lost in a novel situation, which will reappear upon greater experience with the new medium of expression.

8. (d) This must be recognized as requiring a simplification of an expression by means of the Rule of Signs and the Index Law, and concerning the b factors, the x factor, but not the "(d)". The signs really belong to the understood coefficient of 1, and as such, when multiplied,

give a positive value to the answer. The b^5 and b^2 mean that b is taken as a factor 7 times in all, and thus give b^7, later to be multiplied by the x factor.

Results:	City	Town	Rural	Totals
Correct	41	44	51	136
Incorrect	91	95	89	275
Not attempted	18	11	10	39

The most common mistake noted was that the candidates did not realize that the b factors could be, and must be, combined into a single power of b. This error occurred in about 15% of all answers. Signs were wrong in about 10% of the cases. The "x" factor was omitted by some, and the "(d)" was put in as a factor, by others. Another fairly frequent mistake was subtraction of the indices.

This question depended to a large extent upon recognition of the application of the Index Law. Apparently, its use is not too well known. On no paper did a student put the factors down and later combine them. This, of course, should not have to be done, but it is a safe, if slow, method of arriving at an answer.

8. (e) The brackets indicate that the whole of the expression must be cubed -- taken as a factor three times. It means that "-5" must be cubed, and the "d" cubed also. A student was expected to realize that 5^3 was to be expanded but that the d^3 was to be left in that form -- owing to the very nature of the numerical and literal coefficients.

Results:	City	Town	Rural	Totals
Correct	59	56	68	183
Incorrect	84	91	80	255
Not attempted	7	3	2	12

About 8% of the pupils had trouble with the signs and arrived at "$125d^3$". Others had the sign correct, and had "d^3", but took the value of "5x3" as the coefficients. About 6% were so occupied with cubing "-5" that they neglected to cube the "d". Almost 10%, exclusive of the above, had the wrong power of "d". Another 5% had other than the above-mentioned coefficients. About 15% had miscellaneous incorrect answers which indicated that the concepts were totally wrong.

This is a test of knowledge of several concepts -- use of brackets, cubing a number, and cubing a literal factor. Also there is the necessity of doing not one, but all, correctly. Constant drill, in the mental field would eliminate much of the uncertainty and confusion in regard to these principles.

8. (f) This is related to Part (e) above. The factors within the brackets are to be taken twice, the number to be expanded, and the literal factors expressed as powers. The whole is to remain negative owing to the position of the -1 -- it is not to be squared.

Results:	City	Town	Rural	Totals
Correct	25	26	32	83
Incorrect	119	118	115	352
Not attempted	6	6	3	15

Nearly 33 1/3% of all students made their mistake in signs --

gave "$16s^2t^2$" as the answer. 7% gave "$-8s^2t^2$" as their answer -- everything correct but squaring the "4". 8% had "plus or minus 16st". There were many miscellaneous wrong answers having powers as high as the fourth, and coefficients as high as "256".

The percentage of correct answers was much lower in this question than in Part (e) above. The greatest trouble was in interpreting the role of the sign in relation to the expression. The only criticism that can be given is the one made in connection with the related question mentioned -- only much more drill, and preferably mental drill, will give students confidence in handling these new concepts.

8. (g) This division of a polynomial by a monomial requires much the same types of skills and concepts that we have seen before -- a knowledge of division, Rule of Signs, cancellation of coefficients, and cancellation of literal factors.

Results:	City	Town	Rural	Totals
Correct	54	59	66	179
Incorrect	85	77	78	240
Not attempted	11	14	6	31

These figures stand for initial errors only -- no attempt was made to tabulate errors subsequent to where the pupils first went wrong. Errors in signs occurred in about 20% of all answers. About 16% of all answers had errors in regard to literal factors. A large percentage -- over 7% -- gave an answer which contained a number of terms other than three.

Evidence keeps increasing as one goes through all the answers

in a set of papers, that the fundamental concepts -- signs, indices etc., -- are not mastered, even by a majority. Mechanical errors may be made because of carelessness, but errors in signs are made because the rules are not learned. Pupils don't know them sufficiently well to be able to guarantee results.

Value
6 9. If $3/5\ x - 3\ 1/3 = x/2 + 4$, what must be the value of x ?

(Show all computations in the space immediately below.)

9. An equation of this nature demands comprehensive knowledge of the principles of solving equations, and the mechanical operations that make these principles operative. Two methods are in vogue in working this question: clearing of fractions first, or, transposing first, and adding and subtracting the fractional values.

If the pupil chooses to clear of the fractions he must find a common multiple of the denominators, and then use this to multiply the algebraic expressions, mixed numbers, proper fractions, and whole numbers. Since some of these are signed numbers he must also be able to multiply signed numbers according to the conventions. If this is done correctly, a fairly simple equation results which must be solved by transposition and other operations.

However, it may be done by transposing first, and using the fractions and mixed numbers as such.

Results:	City	Town	Rural	Totals
Correct	50	58	40	148
Incorrect	82	74	92	248
Not attempted	18	18	18	54

Upwards of 20% of all pupils made mistakes in the operations with fractions. About 16%, using the method of clearing of fractions,

multiplied three of the four terms of the equation, but neglected to treat the fourth in a similar manner. Errors in transposing accounted for a small percentage of the errors. Some four people actually copied the question incorrectly in the proper space and then proceeded to solve their equation correctly.

Some of the work examined showed that the pupil had no idea as to what to do -- the most inconceivable things were attempted. Much of the work was untidy in the extreme -- no system and no effort at neatness could be discerned.

With regard to mechanical errors -- whole numbers and fractions -- they seem to be, like the poor, always with us. One would say that the grades before Grade IX are the places where these fundamental operations should be mastered. A student cannot give undivided attention to the concepts of Algebra if he is preoccupied with multiplying a fraction by a mixed number. If necessary, the courses in Mathematics in the grades below Grade IX should be reduced to something approximating the bare essentials -- at least for the lower third of the class. More rigid segregation of students in the lower grades, and limiting of the course of study would enable them to become masters of a few trades rather than merely having a speaking acquaintance with many.

With regard to the principles of solving equations, it might be deemed wise to admit that all pupils at this level should be regarded as having limited ability, and thus are unable to master various methods of solving equations. Instead of leaving a pupil with the hazy notion that either transposing or clearing of fractions might be done first, a pupil should be told, unreservedly, that whenever fractions occur, they must be cleared at the start. It should not be left to the judgment of

the pupil. This may be thought of as narrowing the scope of a pupil's ideas. It is. However, it is contended that it will give the weaker portion of a class more power and confidence, and it will only postpone the day until the superior part of the class will be able to go on to wider concepts in a much more natural way.

Values

10. Write the equation that might be used to solve each of the following problems. Do not solve the equations.

3 (a) In a certain city the taxi fare is 30 cents for the first mile and 20 cents for each additional mile. How far did a passenger travel if his fare was $1.70 ?

Equation is : ☐

3 (b) Four-fifths of a certain number is 120 more than half the number. Find the number.

Equation is : ☐

3 (c) The sum of three consecutive numbers is 57. Find the smallest of the three numbers.

Equation is : ☐

10. As we enter problem work, correct silent reading -- interpretation of the situation -- becomes of paramount importance.

In order to have problem solving ability, a pupil must acquire, and make his own, some definite, formal set of rules which guide his thinking about, and comprehension of, a problem. Such a set is the one advocated in the authorized textbook for Grade IX Mathematics, "Mathematics for Today" -- LaZerte and Betz, on pages 74 and 75. These steps should be known, understood, and followed implicitly.

This implies that when a pupil is asked for an equation that might be used to solve a problem, he shall represent the unknown by a letter, express any necessary relationship required in the problem, and then write an equation showing the relationship existing between the

algebraic quantities and other pertinent material. Then, in regard to this particular examination question, the correct, unsimplified equation should be given as the answer.

No provision, as to space on the answer paper, was made, or credits given, when the first two steps were carried out.

This question is further evidence that one method, and one method only, should be taught for doing one type of work at this level. Unless the problem is read to the point of complete comprehension, the unknown expressed, and any necessary relationships expressed, the problem solving ability of Grade IX students is bound to suffer.

Results:		City	Town	Rural	Totals
(a)	Correct	6	9	3	18
	Incorrect	123	124	143	390
	Not attempted	21	17	4	42
(b)	Correct	68	82	68	218
	Incorrect	64	61	70	195
	Not attempted	18	7	12	37
(c)	Correct	82	76	82	240
	Incorrect	53	63	62	178
	Not attempted	15	11	6	32

Special results:

(a)	$20x + 30 = 170$	23	21	25	69
	$20x + 30 = 1.70$	23	30	34	87
	$20(x-1) + 30 = 1.70$	2	4	1	7
(c)	$n-1+n+n+1 = 57$	6	0	1	7

$n-2+n-1+n = 57$	1	3	3	7

With regard to the special results tabulated above, there were partial credits awarded in Parts (a) and (c). The equation "$30 + 20(x-1) = 170$" received full marks, with one mark taken off if the numbers were written in the first member in cents and in the second member in dollars and cents. The equation "$30 + 20x = 170$" was awarded two out of the three marks. There was a further deduction of one mark if the units of cents and dollars were used.

10. (a) About 16% of all students answered the question with no equation being given, or if an equation was given, there was no unknown in it. About 20% gave evidence that they were completely at sea -- they had no idea what was wanted or what could be done. A few gave the answer, "8 miles" but received no credit -- they didn't do what was asked although they were able to solve the problem.

10. (b) In Part (b) the errors were fewer in number. About 16 2/3% showed that they had interpreted the covering question and part of the problem correctly, but only had one side of the equation correct. Various types of errors occurred in the second member. Within a range of from 10% to 20% in the three areas, the answers showed that neither the covering question nor the specific problem were comprehended to any appreciable degree.

10. (c) In Part (c) the results were even better -- over 50% of the students had the correct equation.

The most common error was to start from "$n+1$". It could be argued that this is not an error -- a series starting from "$n+1$" can be

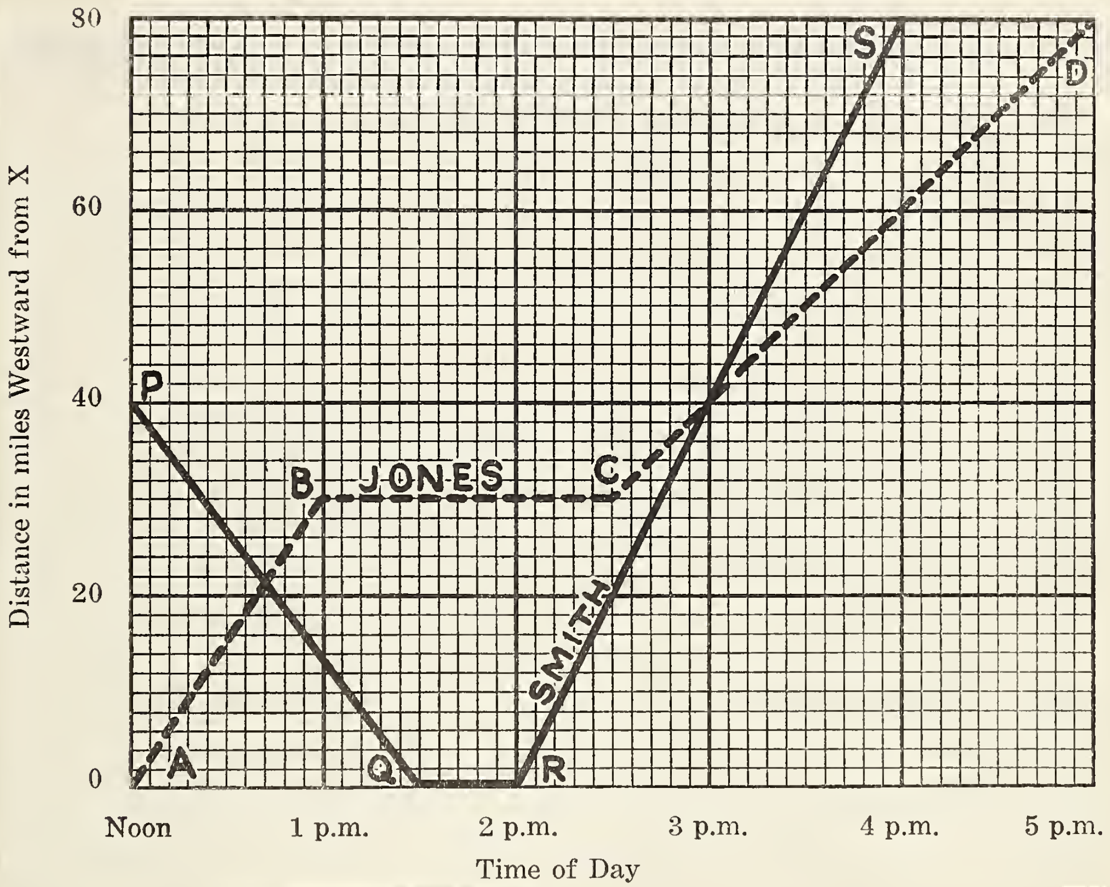
formed to arrive at an equation that might be used in solving the problem. Another common mistake was to use a series of consecutive even or odd integers. Still others, about 10% of the total used x, 2x, and 3x as their series. In a few other cases, where no method was in evidence, no unknown was used, three unknowns were used, or the answer was given.

This question had some unfortunate aspects. There was no space provided for a statement of the unknown, and the necessary relationships. Then from the standpoint of marking, some answers that are correct were not given full credit.

The units in the first part were not properly handled. Students seem to think such things are of no importance. They believe they know how to take 30 from 1.70 and get 1.40. They fail to realize that unless habits of accuracy and care are developed in regard to simple problems they won't be able to handle the more complex problems of advanced mathematics. Any other criticism that might be advanced about the answers of this question revolve around the contention that a formal set method of solving problems should form the basis of all attempts to do questions of this, and a similar type.

Values

11. A highway runs due west from town X to town Y (80 miles). In the figure below, the travel graph ABCD represents the journey of Mr. Jones and PQRS that of Mr. Smith along this road on the same day. Answer the questions given below the figure.



1 (a) Where did Smith start from at noon ? ☐

1 (b) How far did Smith travel between noon and 3:30 p.m. ? ☐

2 (c) Between noon and 1:30 p.m. Smith's speed was ☐ miles per hour

and between 2 p.m. and 4 p.m. it was ☐ miles per hour

2	(d) Jones and Smith met on the road at	p.m.
2	(e) They met	miles from X.
1	(f) How many miles was Jones from Y when Smith left X ?	
1	(g) At what time did Smith overtake Jones?	
2	(h) What was Jones' average speed during the time he was actually travelling?	
2	(i) Mr. Brown left Y at 1 p.m. and motored to X in 3 hours, travelling at the same speed all the time. Draw his travel graph. He met Smith on the highway.......................... minutes after he met Jones.	

11. For a Grade IX student, a graph should be a representation of two variables -- time and distance, in this question. These variables and their units, are shown on the axes of the graph. Each individual space between the lines which carry the numbers has a definite value,and this value has to be understood in order that a graph may either be read or drawn.

Given the above data, any point on the graph tells two things -- a specific time and a specific place. Similarly, if one of these variable factors is mentioned, either a particular time or a particular distance, then the corresponding distance or time can be read from the graph. This is the basis of interpreting the graph. On the other

hand, if the relationship that exists between the two variables is given -- e.g., a person at a particular place at a given time, then the point can be located. This is the basis of drawing graphs. All of these types of work are required in answering the questions based on the above graph.

Rather than tabulate the results in this question, and then go on to discuss various outstanding errors, the actual numbers of the various answers will be given. This will serve the same purpose.

Results:	City	Town	Rural	Totals
(a) Correct:				
"40 miles west of X"	89	102	87	278
"at P"	35	32	43	110
"1/2 way"	9	3	8	20
"at 40","40 miles","40"	5	4	0	9
Miscellaneous incorrect answers	11	8	10	29
Not attempted	1	1	2	4
(b) Correct:				
"100 miles"	55	46	46	147
Incorrect:				
"60 miles"	34	37	21	92
"90 miles"	7	1	7	15
"80 miles"	6	3	7	16
"110 miles"	11	9	9	29
Miscellaneous incorrect answers	32	43	54	129
Not attempted	5	11	6	22

(c) Correct:				
"26 2/3 m.p.h."	44	46	55	145
Incorrect:				
"40 m.p.h."	13	11	10	34
"30 m.p.h."	15	16	9	40
"0 m.p.h."	4	5	3	12
"20 m.p.h."	8	10	12	30
"13 1/2 m.p.h."	3	5	2	10
"60 m.p.h."	6	7	3	16
"27 m.p.h."	5	2	4	11
Miscellaneous incorrect answers	38	31	36	105
Not attempted	14	17	16	47
(c)2 Correct:				
"40 m.p.h."	101	98	95	294
Incorrect:				
"80 m.p.h."	13	12	10	35
"60 m.p.h."	4	0	3	7
"20 m.p.h."	4	7	5	16
Miscellaneous incorrect answers	13	15	25	53
Not attempted	15	18	12	45
(d) Correct:				
"12:42 p.m."	25	21	20	66
"12:45 p.m."	11	5	7	23
Incorrect				
"3 p.m."	98	105	104	307
Miscellaneous incorrect answers	15	18	12	45

Not attempted	1	1	7	9
(e) Correct:				
"20-22 miles from X"	35	29	27	91
Incorrect:				
"40 miles from X"	89	101	92	282
Miscellaneous incorrect answers	19	16	26	61
Not attempted	7	4	5	16
(f) Correct:				
"50 miles from Y"	30	31	33	94
Incorrect:				
"30 miles from Y"	18	20	12	50
"40 miles from Y"	24	30	19	73
"20 miles from Y"	5	3	3	11
"25 miles from Y"	5	5	6	16
"80 miles from Y"	6	11	19	36
"45 miles from Y"	7	4	7	18
"0 miles from Y"	4	1	1	6
Miscellaneous incorrect answers	21	22	13	56
Not attempted	30	23	37	90
(g) Correct:				
"3 p.m."	121	116	112	349
Incorrect:				
"12:45 p.m."	8	3	5	16
Miscellaneous incorrect answers	13	25	20	58
Not attempted	8	6	13	27

(h) Correct:				
"22 6/7 m.p.h."	21	23	26	70
Incorrect:				
"20 m.p.h."	19	8	10	37
"16 m.p.h."	23	18	21	62
"30 m.p.h."	15	8	12	35
"40 m.p.h."	6	7	6	19
"25 m.p.h."	14	14	7	35
Miscellaneous incorrect answers	29	52	45	126
Not attempted	23	20	23	66
(i) Drawing the graph				
Correct:	50	48	57	155
Incorrect:				
(started from X at 1 p.m.)	27	21	24	72
(started from X at Noon)	13	12	13	38
(started from Y at Noon)	5	5	2	12
(not equal speed)	3	3	1	7
(wrong speed)	3	5	3	11
Miscellaneous incorrect answers	9	2	2	13
Not attempted	40	54	48	142
Answering a question				
Correct:				
"6 minutes"	38	41	46	125
Incorrect:				
(gave time)	3	1	2	6
(miscellaneous - 1-150 minutes)	58	58	52	168

Not attempted	51	50	50	151

One outstanding criticism is that pupils did not read each question carefully enough to be sure they comprehended it. Too many people said that Smith started from P at noon. Now this designation of a point by the letter P is not an integral part of the chart -- it is used as an auxiliary method of designating one graph. Again, in Part (d) they did not differentiate between the word "met" and the word "overtake".

Many pupils made mechanical errors in working with data read from the graph. This is very much in evidence in those questions which demand a calculation of the average speed, when the time and distance are read from the graph. Two answers involving speeds also involve fractions, i.e., 26 2/3 m.p.h. and 22 6/7 m.p.h. There was difficulty encountered in handling the values which gave these answers.

In only one question, Part (i), were the pupils required to plot points and draw a graph. Just about 33 1/3% of pupils had the graph correct, and about the same percentage did not attempt to do it. Apparently reading from a graph is easier than drawing it. Their attempts to draw the graph give further evidence that instructions are not read well enough, to enable the students to do the required task.

Two marks were originally assigned to the two requirements in Part (i). However, the graph was ultimately awarded 2, and the question based on the graph was given a value of 3.

Value
9 12. Solve one of the following problems:

Either (Three boys sold 90 tickets for a football game. John sold three times as many as Bob and Bob sold half as many as Dick. . How many tickets did each boy sell ?

or (I bought some 50-cent presents and one-third as many 75-cent presents. If I paid $9.00 for all, how many at each price did I buy ?

GIVE SOLUTION BELOW

12. A problem presents an objective, concrete situation more than any other type of work, with the possible exception of a graph. The first element necessary in the solution of a problem is the reading of it to the point of complete comprehension. Then, if the problem is to be solved by the processes of Algebra, a definite chain of steps, more or less mechanical in nature, yet based upon reasoning, must be undertaken. The steps that are to be followed are: represent the unknown by a letter; express any necessary relationships in terms of the unknown chosen; make an equation using the facts of the question; solve the equation; interpret the root of the equation; and check the result in the original problem. These steps, when understood and applied, give a power of solving problems that is of incomparable value.

Results:	City	Town	Rural	Totals
First optional problem				
Correct in every detail	84	98	101	283
Wrong expression of relationship	27	23	25	75
Miscellaneous	19	10	11	40

Second optional problem				
Correct in every detail	3	6	1	10
Wrong expression of relationship	4	4	1	9
Wrong equation	7	5	2	14
Miscellaneous	2	2	1	5
Not attempted	4	2	8	14

About 18% of the students made their initial mistake in expressing the relationship existing among their unknowns, incorrectly, and consequently had the wrong equation. This occurred most often in the first problem, where the smallest quantity of tickets sold was not represented by the original unknown. In the second problem it arose because the pupils did not realize that the values of the presents were needed in order to write the equation. There were comparatively few mistakes other than this and the number of pupils who didn't try the problem was negligible.

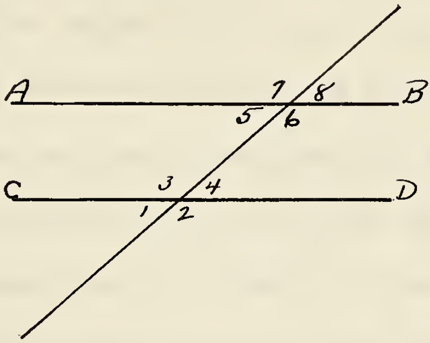
Problem work is the ultimate use to which any mathematical knowledge is to be put. It is most gratifying that while many details of manipulation are not known well, those which are known can be put to practical use in an important field such as problem solving.

CHAPTER IV

ANALYSIS OF GEOMETRY ANSWER PAPERS

Values

1. In the accompanying figure AB is parallel to CD.



1 (a) .... are acute angles;

1 (b) ....and............. are one pair of alternate interior angles;

1 (c) ....and............. are two corresponding angles.

1. This question requires knowledge of three terms, namely, acute angles, alternate angles, and corresponding angles. There is need here, as in all subsequent questions, that the question be read to the point of complete comprehension of what is given, and what is required as the answer or answers.

Results:		City	Town	Rural	Totals
(a)	Correct	116	111	116	343
	Incorrect	33	37	32	102
	Not attempted	1	2	2	5
(b)	Correct	119	125	125	369
	Incorrect	31	25	25	81
	Not attempted	0	0	0	0
(c)	Correct	122	123	114	359
	Incorrect	28	27	36	91

Not attempted	0	0	0	0

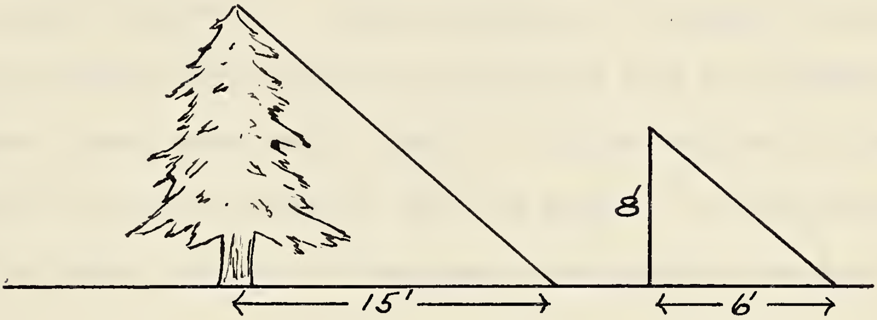
The common mistake in Part (a) was that the pupils did not give the four possible answers. The students apparently knew the meaning of the term "acute angles" but had no way of knowing that the mark was to be awarded on the basis of all the angles being mentioned. There is nothing in the wording of the question to indicate that four answers were required. In Parts (b) and (c) the students who had the answers incorrect revealed that they were not familiar with the meaning of the terms. In their answers, some were prone to put one pair of angles in each blank space when it distinctly said that one pair of angles was all that was wanted.

One would say that these terms are so fundamental in Geometry that it is hardly conceivable that there should be any mistakes at the end of the Grade IX year. The percentage of students giving incorrect answers indicates that many a teacher errs when he takes for granted that any of these terms have been learned perfectly by his students. Again, these same students may have known definitions of the terms but were not prepared to recognize them in practice. A term should only be learned in conjunction with the situations in which it is found or used.

Value

4 2. The upright pole and the tree cast their shadows at the same time.

The tree is.............ft. tall.



2. Basically, the solution of this question depends upon the recognition of similar triangles. The data necessary to recognize the triangles as similar are given in the statement introducing the question. From this point, in order to take the second step, the concept of a simple proportion is essential. The unknown and the data may be related in an equation according to the Rule of Three, or the constant factor in the ratio of the sides of one triangle be determined, and this factor applied to the given side of the second triangle. The first method necessitates working an algebraic equation, either $x/15 = 6/8$ or $x/8 = 15/6$. The second procedure requires the division of 15 by 6 to determine the ratio, 2 1/2. The side 8 ft. is then multiplied by this constant.

Results:	City	Town	Rural	Totals
Correct	69	82	73	224
Incorrect	70	62	68	200

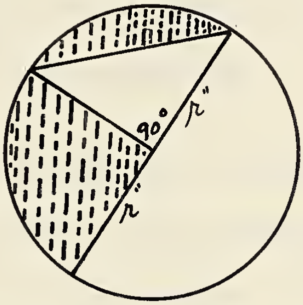
Not attempted	11	6	9	26

The greatest percentage of wrong answers, varying from 16% to 25%, in the three groups, occurred because the pupils did not know the principle of simple proportion. They recognized, or assumed, that the triangles were similar, but instead of dividing the 8 by 6 to determine the ratio factor, they subtracted 6 from 8. This difference of 2 was applied to the 15 to get an answer of "17" (in feet). In many instances the work was not shown, and, as a consequence, the mistakes could not be analysed.

The topic of similar triangles together with its application to finding inaccessible distances is taught in Grade IX. This was reasonably well done as mistakes in this concept were comparatively few. But the mechanics of working the questions apparently were not sufficiently stressed. This is evidenced by the numbers of students who gave the answer, "17 feet", as noted above, and by others who gave improbable answers. This type of question is one in which clear observation and a "sense of proportion" or judgment is almost a necessary adjunct to any geometrical concepts. Just about 50% had the answer incorrect, and this seems too many in view of the nature of the question. Unlimited practice, together with instructions to use common sense, would remove some of the difficulty.

Value

5 3. The total area of the shaded portions of the figure below is..................square inches.



3. It would seem reasonable to assume that Grade IX pupils know how to find the area of a circle, and the area of a triangle. It might, however, be questioned, whether they have a clear concept of a literal value of the radius, and whether they would recognize that the altitude of the triangle was equal to r. Be that as it may, the main concept here seems to be that the area of the shaded parts (one of them being a shape that can easily be found, and the other a shape with which they are unfamiliar) is to be found most conveniently by the simple expedient of taking the unshaded portion, which can be found with the given data, away from the whole, whose area can be calculated with ease.

Results:	City	Town	Rural	Totals
Correct	13	12	11	36
Incorrect	83	87	86	256
Not attempted	54	51	53	158

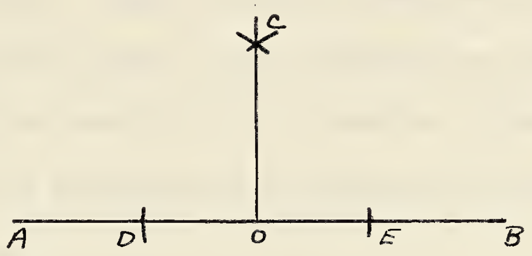
Many students, over 16 2/3% of the total, gave a numerical value as their answer. Some 10% of the pupils had the answer correct

except for recognizing that the altitude of the triangle was r -- they called it "b" or "a". There were a few mechanical errors made in simplifying an originally correct expression. The nature of the errors in the rest of the wrong answers could not be found, as the work shown was too meagre to be of any value in this connection.

There seems to be a major obstacle to correct thinking about a problem, at this, the Grade IX level, in the fact that the use of symbols, representing numbers, forms part of the problem. This is evidenced by the prevalence of answers in numbers only. The principle that a constant such as "pi" may be left, as such, in an answer is not generally accepted by the student. An answer with "pi" as a factor, is as good, if not better for some purposes, than one in which 22/7 has been used. Some students wasted valuable time, and some even introduced errors into their work by simplifying, what, to them, must have been a too-complex answer.

Then too, the sustained chain of reasoning that the shaded portions are in one-half the circle (which must be found) and that the required answer is found by taking the area of the triangle from one-half the circle, proved too much for the great majority of students. To a mature student all this seems like a very natural procedure, but the concept has not been acquired by our Grade IX students. The elements of thought required in the correct solution seem to be known, but insufficient practice in this type of work results in a sense of doubt and confusion about the whole process.

Value
1 4. When the perpendicular to AB was drawn below, the point of the compasses was placed first at ....



4. Since this question says, "When the perpendicular to AB was drawn.....", pupils should immediately call to mind the two methods of drawing perpendiculars, namely, from a point within the line, and from a point without the line. Enough information was given in the explanatory data of the question to obviate any confusion between this figure and the one for the bisection of a line. The appearance of the intersecting arcs at C should mean that it was found -- not given. Thus, by a process of elimination, if no more direct method had presented itself, the correct answer should have been arrived at -- that the point of the compasses was first placed at O.

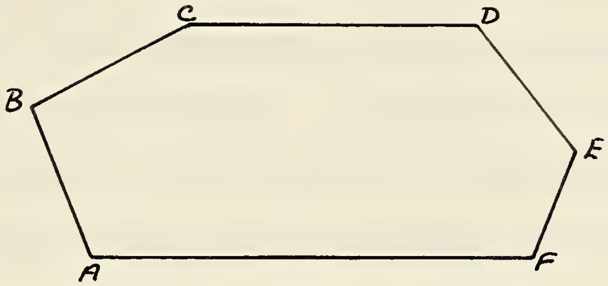
Results:	City	Town	Rural	Totals
Correct	68	65	51	184
Incorrect	82	85	99	266
Not attempted	0	0	0	0

"A" or "B" were given in 25% of all answers. "A and B" occurred in over 13% of all cases. "C", "D", "E", and "D and E" made up the balance of the incorrect answers.

From the students' viewpoint, this question was apparently so simple that no thought need be put on it. It was attempted by all the students, but more than 50% of the answers were incorrect. The question was not carefully read. Pupils wouldn't have given two answers for a question which said "the point of the compasses was first placed at....". Lack of a comprehensive reading of the question, together with disregard of the evidence of the positions of the arcs etc., were responsible for the very poor results.

In this question, as well as in others, there is indication of a serious lack of a desire to do a task thoroughly. The method of verifying a construction by doing it, would be applicable here. If a student had gone to the trouble of constructing the figure, the doubt as to which construction was being attempted, and as to what was given or required, would have been removed. All this would have been rendered unnecessary if the concept had been on the mental level, as opposed to the manual level.

Value
3 5. The sum of the angles in the accompanying figure is .... degrees.



5. Two methods are available in working this question: the formula, S = (2n-4) right angles, could be used; or the figure could be divided into triangles, and the sum of the interior angles of the figure found by using the fact that there are two right angles in the sum of the three angles of any triangle. Another method, and a very natural one, apparently in Grade IX is to measure the angles and find the sum.

Results:	City	Town	Rural	Totals
Correct	60	66	52	178
Incorrect	87	81	95	263
Not attempted	3	3	3	9

In 7% of all cases pupils divided the figure into six triangles by selecting a point within the figure and joining this point with the corners of the figure -- but they didn't subtract the 4 right angles at the centre from the total of 1080 degrees. The wrong answer, "360 degrees", appeared on 20% of all papers, and no reason was seen for the nature of the error. Of the four hundred and fifty papers under consideration, forty pupils measured the angles and added -- and only one

got the right answer. Some students found the number of degrees in each interior angle of a regular six-sided figure. The miscellaneous wrong answers varied from "30 degrees" to "6200 degrees".

Geometry in the grades prior to Grade IX is mainly concerned with construction and measurement. In Grade IX an attempt is made to introduce the students to the other aspect of Geometry, namely, that because of a carefully built up body of knowledge, we can know, or deduce, facts about figures without the use of instruments.

The two methods of gaining knowledge must still be used, but it is necessary to judge as to which method is applicable in any particular situation. It may be too much to expect that pupils would gain mastery of such a principle as this in one year, but it should not be too much to expect that they should learn the principle and be on the alert to apply it.

In this question there are the few who are prone to make mechanical errors and to make errors of omission, such as not taking 4 right angles from the sum of the angles of the 6 triangles they constructed. These people will, no doubt, overcome this weakness by continued practice and a desire to excel. There are those, however, who are going to need more of that same practice and that same desire to improve. Pupils who put down answers varying from 30 degrees to 6200 degrees are not using any critical judgment. Perhaps they advanced from the stage of construction and measurement too rapidly.

Value
6 6. In the floor plan of a hall drawn to scale (3/8" = 10 ft.) a room is 2 1/4 inches long and 1 1/2 inches wide. What is the actual floor area ?

☐ sq. ft.

6. This question demands a rather comprehensive chain of reasoning. The simplest way to consider it is: change each of the scale dimensions to actual dimensions, then find the area. This necessitates dividing each scale dimension by 3/8 and multiplying the answer by 10 feet. This procedure assumes that all students will recognize that area is to be found, and will know how to find it. Involved here, too, is a knowledge of mechanical operations with fractions and ability to make ordinary arithmetical calculations.

Results:	City	Town	Rural	Totals
Correct	89	94	91	274
Incorrect	60	54	55	169
Not attempted	1	2	4	7

A few people left their answer as "24 square feet". About 3% showed, in their work, that they knew the principles underlying this question, but made mechanical errors. Others used 8/3 x 10 once in multiplying by the dimensions; others didn't invert the 3/8 when dividing the scale dimensions by 3/8; some gave the formula: "A = lw"; and one person found the perimeter. Answers ranged from "9 square feet" to "4246 1/3 square feet".

There is a formal method of problem solving applicable in this

question. It is the so-called Unitary Method of Problem Solving. It has an extremely wide application in the hands of a person who once masters it.

e.g. 3/8" on drawing represents 10 feet
1" on drawing represents 8/3 x 10 feet
2 1/4" on drawing represents 9/4 x 8/3 x 10 feet = 60 feet

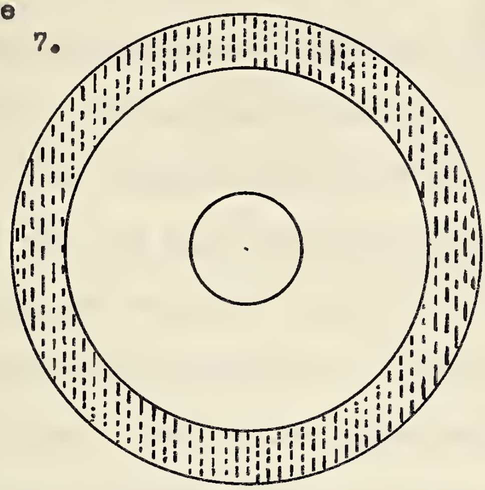
It was assumed, prior to the introduction of the new Programme of Studies into the Alberta Schools, that this method was introduced early in school -- at about Grade III (in its very simplest form and application, of course) and was mastered at the end of the Grade VII level. It is not now used to the same degree of intensity as it formerly was.

Its mastery was attained through much meaningless practice, but, in a large measure, insight into the operations involved, was gained somewhere in the learning process. Comprehension of this method gave a real power to the person who had gained it. Easy applications, and very involved applications, were then made with equal facility. There was no sense of confusion, or doubt or uncertainty. In this example, there would be no question whether to divide by 3/8 or multiply by 3/8; whether to multiply by 10 once or twice.

It is contended that most Grade IX pupils are able to handle questions such as the one under consideration if, and only if, the numbers are simple whole numbers. The fact, that fractions are involved, complicates it to the point where doubt enters, and although pupils attempt the question in good faith, the necessary power to handle difficult numbers is lacking. More persistent drilling in fundamental types

of reasoning, such as the Unitary Method or the Rule of Three, would give the necessary power once they were mastered, over a very wide range of problems. Grade IX pupils need this sense of power and security.

Value
6 7.



The radii of the three concentric circles in the figure on the left are 3",9" and 12" respectively. The area of the outer shaded ring is

..................times that of the innermost circle.

7. The solution of this question involves knowledge of a very well known formula, the area of a circle, a rather intricate relationship of areas, and some involved work in vulgar fractions -- all based upon careful reading and interpretation of the question and figure. Since it was attempted by all but 4% of the students, it may be assumed that 96% of them knew how to find the area of a circle -- perhaps the 4% knew this too. Careful reading and understanding of the instructions would lead to the idea that the area of the ring was involved -- and not the area of the largest circle, directly. Then, the area of the ring had to be divided by the area of the inner circle. It is possible, of course, to do the question by using "pi" as a factor which will cancel out, without using its value of 22/7.

Results:	City	Town	Rural	Totals
Correct	22	23	15	60
Incorrect	123	122	128	373
Not attempted	5	5	7	17

The percentage of correct answers was quite low. Some of the answers classified as incorrect received partial credit for the part of the work that was done correctly. Upon examining the errors, it was found that slightly over 5% of the students had knowledge of the right principle, but made mechanical errors. Over 22% of students divided the area of the outer circle by the area of the inner circle. About 4 1/2% found the areas of the three circles but couldn't proceed beyond that point. 26% of the papers had an answer of "4" indicating that students had divided the radius of the outer circle by the radius of the inner circle. In a few cases the radius was not squared in finding area, and in others the "pi" was squared.

This question serves to emphasize the contention that pupils cannot see the relationship of a familiar principle to a rather complicated whole-situation.

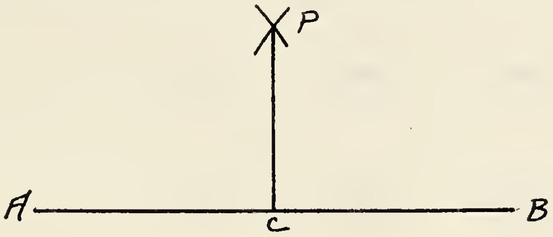
Perhaps finding the area of the ring by taking the 9" circle from the 12" circle is the difficulty. However, it would seem a reasonable procedure since absolutely no other method, for Grade IX students, presents itself -- and the pupils have worked examples using the principle. Superimposed upon this are some rather complicated and perhaps tedious manipulations of fractions, which should not, of themselves, obscure the method of working the question. Altogether, one would be inclined to say that it was a combination of the many difficulties rather than any individual one, that led to the poor results.

This difficulty may be blamed on carelessly done reading or general lack of concentrated effort, or both. Antipathy to involved mechanical work might lead to the feeling that any answer should be put

down rather than be bothered further with the question. Carelessness alone can be blamed for the mechanical errors in those answers in which the principle was correct.

Values

8. The instructions and statements given below are wrong in several places. Name three errors and tell in each instance wherein the error lies.
(Construction 1). To bisect a given line-segment. Suppose AB is the line-segment to be bisected.



Step 1. Take A as centre.

Step 2. With any radius, draw an arc of a circle.

Step 3. Then using B as centre and the same radius draw an arc of a circle.
Step 4. Let this circle cut the first arc at P.

Step 5. Join P to the middle point of AB.

2 Error (1)

2 Error (2)

2 Error (3)

8. This question purports to test how meaningful instructions, with regard to any construction, are. In approaching this question, from the students' viewpoint, the first concepts are to be found in the instructions at the first of the question. Secondly, the steps in bisecting a line-segment must be known. And, what is more important, the reasons for these steps must be known. The instructions in the preliminary part of the question must be carried out in the light of the knowledge of the construction proper. If "any radius" is taken, the arcs won't necessarily intersect. Only intersecting arcs are required--

not whole circles. There must be two sets of arcs, one set above and one below the line-segment. The points where the two pairs of arcs intersect must be joined -- it must not be assumed that the mid-point is known.

Results:	City	Town	Rural	Totals
Error (1)				
Correct	59	71	81	211
Incorrect	88	78	66	232
Not attempted	3	1	3	7
Error (2)				
Correct	49	55	47	151
Incorrect	96	92	99	287
Not attempted	5	3	4	12
Error (3)				
Correct	52	44	50	146
Incorrect	89	101	91	281
Not attempted	9	5	9	23

Part marks were awarded when the error was corrected but when it was not properly placed as to the step in which it occurred. These were arbitrarily classified as correct by the investigator, because the Geometry was properly understood.

It should be safe to say that nearly all Grade IX students are able to bisect a line, but, from the results obtained from an examination of the answers to this question, it would seem to be an unreasoning, memory work that gives this ability. Since Geometry is designed partly

as a course in reasoning, an opportunity missed, to emphasize this nature of the subject, is to be regretted.

This is one type of question that seems to be extremely valuable from the standpoint of reasoning. Pupils need practice in detecting examples of reasoning based on instructions that are too limited or are omitted entirely, and unwarranted assumptions. The results would indicate that this type of work is not stressed sufficiently. Since it is a very valuable training one would be inclined to say that it should receive extra attention.

Value

3 9. A triangle has sides that are 16 ft., 18 ft., and 24 ft. in length. Is the triangle acute-angled, right-angled or obtuse-angled ?

Give reason for your answer. ☐

9. This question is worked on the basis that when the squares of two sides of a triangle are together greater than the square on the third side, an acute angle occurs between the first two sides. If the sum is less, an obtuse angle results; and if the sum is equal to the square on the third side, a right angle results. Then, the triangles are acute-angled, obtuse-angled, and right-angled respectively. If there is any other reason for the correct answer at this stage of the work, it would be that figures drawn to scale reveal three acute angles. Of course, pupils must understand what is meant by an acute-angled triangle, an obtuse-angled triangle, and a right-angled triangle.

Results:	City	Town	Rural	Totals
Correct (with reason)	15	13	17	45
Correct (no reason)	12	39	36	87
Correct (wrong reason)	30	7	7	44
Incorrect	88	85	85	258
Not attempted	5	6	5	16

Nearly 10% called it right-angled, and gave no reason. Another 10% called it right-angled because they had drawn the triangle to scale and found a right angle in it by inspection. 9% called it right-angled

for an inconsistent reason, such as that "$16^2 + 18^2 = 24^2$". Almost 20% called it "obtuse-angled" because there was "one angle greater than a right-angle", or "scale drawing has an obtuse angle". Some few people answered the question by saying: "Any one of the three", "none" or "can't tell".

The large number of incorrect answers, and the wide assortment of reasons showed that the concepts underlying the securing of the right answer were not held by many, and not mastered by many more. In this question there seemed to be a disposition to make a scale drawing, but there was the lack of the ability to do it accurately enough to be able to draw a valid conclusion from it.

Value

10. In the theorem "The opposite sides of a parallelogram are equal",

2 the hypothesis is..

2 the conclusion is..

10. Basically, this question requires a knowledge of the two terms, hypothesis and conclusion, and knowledge of a particular theorem. There are two difficulties: the two parts are mixed in the theorem as it is stated; the hypothesis presupposes a knowledge of the definition of a parallelogram. Beyond this, it must be understood what is required in the answer -- a particular hypothesis and conclusion, rather than definitions of the terms themselves, must be given.

Results:		City	Town	Rural	Totals
(a)	Correct	44	40	38	122
	Incorrect	82	78	82	242
	Not attempted	24	32	30	86
(b)	Correct	34	40	30	104
	Incorrect	86	85	85	256
	Not attempted	30	25	35	90

About 11% of the students divided the statement after the word "parallelogram". About 2% gave definitions of the terms. 9% put the conclusion in Part (a). 8% put all the statement in Part (a) and 4% put it in Part (b).

This question requires either a thorough understanding of the comcepts underlying theorem, hypothesis, and conclusion, or it demands thorough memory work -- neither of which was done by the majority. If Geometry, as a study in logic, is to be used in preparation for logical thinking in the realm of every-day living, then the materials of Geometry must be learned in order to be able to reason logically in that particular field. If this material -- definitions, constructions, theorems etc., -- is not learned, there can be none of the desired thinking in the realm of Geometry and it may be presumed, no carry-over into the realm of every-day experiences.

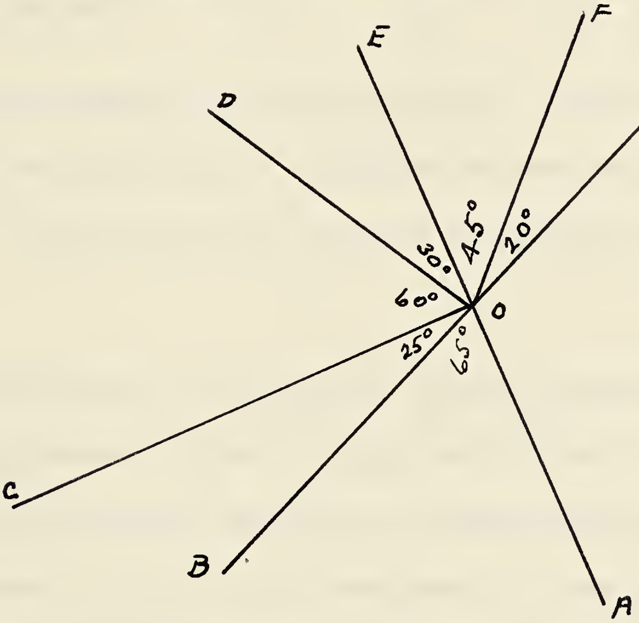
Values

11. In the accompanying figure,

2 (a) Two vertically opposite angles are∠ ......and ∠......

2 (b) Two complementary angles are ∠............and ∠......

2 (c) Two supplementary angles are∠............and ∠......



11. Question 11 is a check of knowledge of definitions of three terms. The figure contains all three types of angles which serve to obscure any individual type. There is extraneous matter also, in the form of right angles and obtuse angles. In answering the question, it is necessary that a pupil be able to name angles correctly.

Results:		City	Town	Rural	Totals
(a)	Correct	76	76	78	230
	Incorrect	74	73	69	216
	Not attempted	0	1	3	4
(b)	Correct	87	86	77	250

	Incorrect	63	61	72	196
	Not attempted	0	3	1	4
(c)	Correct	73	85	79	237
	Incorrect	73	60	69	202
	Not attempted	4	5	2	11

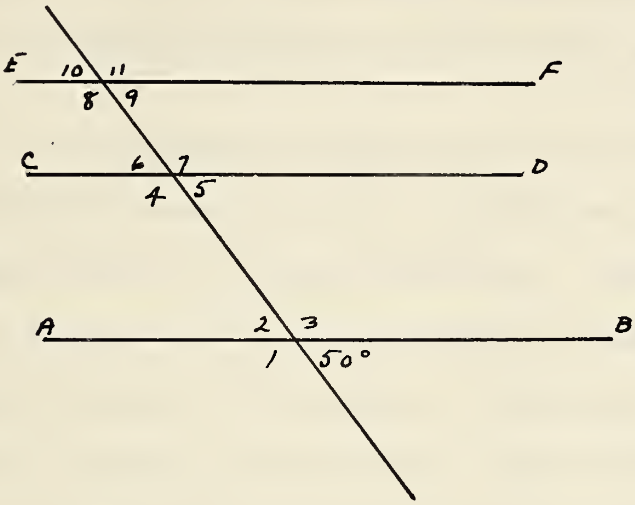
Some errors were made in naming angles. The other mistakes indicated that the pupils didn't recognize the particular angles required in a figure in which there was material irrelevant to the questions asked.

The complication of a figure by putting material in addition to the minimum requirements creates a situation with which pupils cannot cope. The definitions here required are comparatively simple, but the angles to be named, in the light of knowledge of those definitions, are by no means obvious. Teachers might be well advised to give additional emphasis to this phase of the work.

As has been noted before, there seems insufficient effort or drill given to establish firmly a basic concept. The case in point centres around the naming of angles. Three methods are taught, namely: using three letters, using one letter, and using a number. It is contended that sufficient time is not spent on mastering any one of these, because, when all pupils are obligated to use one method they are unable to do it successfully. Some tried to get over the difficulty by numbering the angles, others spoke of "the angle O" not realizing that O designated a point -- the common vertex of at least seven angles. It is believed that if one basic method, that of requiring three letters to

name an angle, had been used exclusively, the simpler methods would be understood in figures where there was no possible ambiguity.

Value
2 12.



The lines AB, CD and EF are parallel.

(a) angle 4 =

........degrees;

(b) angle 9 =

........degrees.

12. To arrive at correct answers in this question, a pupil must know the principle that sizes of these angles were to be found by calculation rather than by measurement, and must have a knowledge of certain definitions of various kinds of angles, and the relations that exist between them, namely, vertically opposite angles, supplementary angles, corresponding angles about parallel lines, and alternate angles.

Results:		City	Town	Rural	Totals
(a)	Correct	124	117	114	355
	Incorrect	26	32	36	94
	Not attempted	0	1	0	1
(b)	Correct	135	131	132	398
	Incorrect	15	17	17	49
	Not attempted	0	2	1	3

In Part (a) about 9% of all students said that angle 4 was

[illegible]

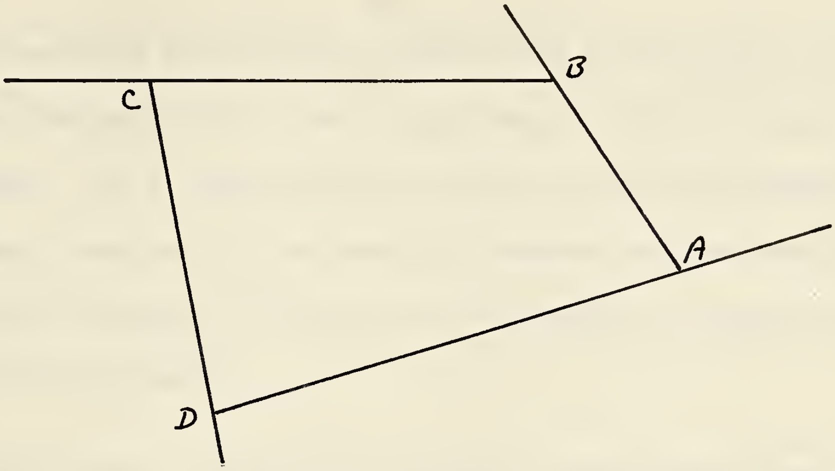
[illegible]

[illegible]

"50 degrees". The range was from "angle 7" and "7 degrees" to "180 degrees". In Part (b), 3% said angle 9 was "130 degrees". The range was from "10 degrees" (presumably angle 10 was meant) to "180 degrees".

The outstanding fact noted in this question, other than the very satisfactory number who had it correct, was that 10% of the students gave answers that are obviously impossible. Angle 9 is acute and couldn't be 130 degrees whatever else it might be, and angle 4 is obtuse, and consequently couldn't be 50 degrees. This lack of ability, or desire, to go beyond some hazy conception of a term or theorem, and bring the question into the light of common sense, is a very discouraging aspect of the work. In this question, the percentage who couldn't or wouldn't bother doing it is amall, but it is much greater in some other questions in this examination paper.

Value
3 13.



Find the sum of the four exterior angles A,B,C and D shown in the figure.

13. The finding of the sum of the exterior angles of a polygon involves much knowledge of Geometry and of some general relationships. The figure must be divided into triangles, or the formula, S = (2n-4) right angles, applied, and the sum of the interior angles calculated. The concept of all that is implied in the term a straight angle, must be known in order to find the sum of the interior angles together with one exterior angle at each corner of the figure. Finally, it must be understood, that the exterior angles may be found by taking the sum of the interior angles from the previously mentioned sum of the interior and exterior angles. Again, it should be realized that the sum of the angles is to be calculated rather than measured.

Results:	City	Town	Rural	Totals
Correct	76	75	65	216
Incorrect	61	56	65	182

Not attempted	13	19	20	52

The majority of the wrong answers were "540 degrees", "570 degrees" and "720 degrees". Some pupils measured the four angles -- with no success. The range in wrong answers was from "45 degrees" to "1440 degrees", showing that not even the haziest idea had been gathered as to the size of angles, or else, that the formula was used incorrectly and quite without judgment.

There seems to be too few pupils who have mastered the principles that we determine quantities by calculation rather than by measuring, or that we can substitute in a formula to achieve the desired result. These concepts are not beyond the comprehension of Grade IX pupils, and should be mastered by means other than taking away their instruments when examinations are set -- as some teachers have advocated in order to curb the measuring tendencies.

The relationship inherent in the fact that the exterior angles are found by subtracting the interior angles from the total of both, was the cause of some difficulties. This is a relationship between a whole and its parts, which, it has been noted, gave trouble on other questions on this paper. More and wider application of the principle would appear to be the method of eradicating such mistakes.

It might be pointed out that the angles B and D of the figure were not exterior angles, and were not the ones the pupils were required to use in their calculations. Whether this led to mistakes on the part of the pupils or not, is not known.

Value

3 14. If the angle between the equal sides of an isosceles triangle is 42 degrees, what is the size of the exterior angle at the base ?

degrees.

14. A chain of reasoning, involving geometric knowledge as the material, is required in the solution of this question. The given angle must be recognized as the odd angle in the isosceles triangle. This, with the knowledge of the theorem which states, "The sum of the interior angles of a triangle equal two right angles", would enable the students to calculate the sum of the interior angles at the base. Since the angles are equal, each may then be calculated. The angle so found is to be taken from 180 degrees to give the desired answer -- the size of the exterior angle at the base. In addition to the chain of reasoning mentioned, pupils must have concepts implied by several terms: isosceles, base and exterior angles. Also, there is demanded some elementary subtraction and division.

Results:	City	Town	Rural	Totals
Correct	76	68	70	214
Incorrect	71	82	80	233
Not attempted	3	0	0	3

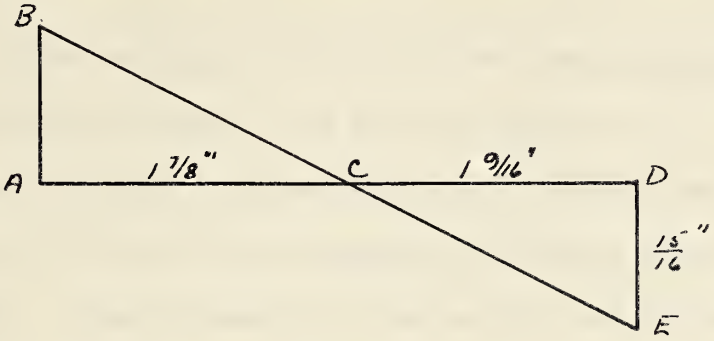
Nearly 7% of all students showed the right method, but made mechanical errors. 20% took the one step, subtracted 42 degrees from 180 degrees, and gave "138 degrees" as the answer. Over 8% did this, then divided 138 degrees by 2, and gave "69 degrees" as the answer.

Over 3% gave "42 degrees" as the answer. The range in the miscellaneous errors was from "3 degrees" to "360 degrees".

In searching for reasons why so many pupils failed to arrive at the correct answer, one might be tempted to say that the concepts required in the solution were not known to the pupils. However, many pupils indicated that they did know these concepts at least in part. It then appears that poor or incomplete reading and interpretation of the question is the main reason for the wrong answers. Nearly 30% of the pupils were correct as far as they went -- but they didn't answer the question asked.

There seems to be very little desire or willingness to sketch a figure or draw it accurately to get help on a question such as this. If measuring is to be of any use, surely it is as a check on theoretical calculation. Pupils should be led to this realization of the use of measurement, rather than to use it where it is definitely not desirable. Only carelessness can be held responsible for 7% of all pupils making mistakes in arithmetical operations -- subtracting whole numbers, and dividing by two.

Value
5 15. The following figure is drawn to scale (1/4" = 1 ft.).



Compute the actual distance represented by AB.

ft.

15. This is another test of knowledge of similar triangles and simple proportion -- as in Question 2 above. In addition to the observations mentioned there, and which are applicable here, the student needed the ability to use a scale, and do mechanical operations with fractions. However, the student was required to assume that the triangles were similar, insofar as only one pair of angles on the figure are necessarily equal. Pupils were expected to assume that the angles at A and at D in the figure, were equal (presumably each was a right angle). This might be considered to be an unwarranted assumption, in view of the fact that the figure is not drawn to scale (1/4" = 1 ft.) -- which the instructions state to be the case.

Results:	City	Town	Rural	Totals
Correct	12	12	7	31

Incorrect	108	102	119	329
Not attempted	30	36	24	90

19% of all pupils said "AB = DE", and proceeded to use the scale, and find AB actually "3 3/4 ft." (15/16 x 4). 16% measured AB on the scale drawing, applied the scale correction and got an answer of "3 1/2 ft." or "3 1/4 ft.", depending on whether AB was measured as 7/8" or 13/16". About 8% used the right method, but arrived at a wrong answer because of mechanical difficulties or errors. Just over 2% had some idea of the Rule of Three, but made a mistake in putting the figures in the wrong places in the proportion. It may be argued that these people didn't know the principle of the proportion. There was a large number of wrong answers but with no evidence as to how they were arrived at.

Obviously, the small percentage of correct answers would indicate that this question was not understood. Too many pupils assumed congruency and showed that they missed the lesson of similar triangles. The principle of proportion based on the similar triangles was not known either. The only way of eradicating these difficulties is to work numerous examples with simple whole numbers before going into the questions involving fractions, which must, however, be done ultimately.

This question indicates that more care should be exercised in stating a question. There should be no possibility of an answer being dependent on an unwarranted assumption. Further, it is believed that the figure should be drawn to scale accurately when it purports to be a scale drawing. If the question is designed to evaluate a pupil's ability to work a proportion involving fractions, then instructions

should be issued that an answer based on measuring will only be awarded part marks. A conscientious student would still be justified in measuring the line in order to check his work. After all, that is what a scale drawing is for.

Value
6 16. A line parallel to the base BC of a triangle ABC cuts the sides AB and AC in D and E. Prove that $\triangle ABE$ and $\triangle ACD$ are equal in area.

16. The first major problem confronting a student attempting this question is that the data be properly interpreted and put into a figure. Then the student must realize that the required triangles had equal parts -- triangles that could be proved equal by the theorem which says that "Triangles on the same base and between the same parallels are equal in area". The part common to the two required triangles had to be added to each of the triangles previously proved equal in area. The axiom that "Equals added to equals give equals" had to be given as the reason that the given triangles were equal. The marking was on the basis of this three-fold chain of reasoning.

Results:	City	Town	Rural	Totals
Correct	24	31	32	87
Incorrect	110	110	109	329
Not attempted	16	9	9	34

In the group tabulated as "Incorrect" were 62 students for the cities, 65 for the towns and 60 for the country, who had part marks. Instead of knowing the theorem that "Triangles on the same base and between the same parallels are equal in area", they made some unwarranted assumptions, either in reading the question, or drawing the figure, and proceeded to prove the two triangles that should have been equal in area only, were congruent. Nearly 20% of all students had only a figure to

show for their efforts in this question. In another 8% of the cases, the figure was the only thing done and it was incorrectly drawn.

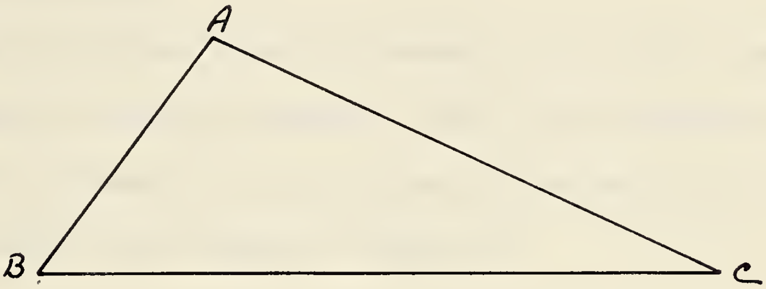
One prime requisite in reasoning is that the given or assumed material must be clearly understood, and the limits imposed by this hypothesis must not be transgressed. Over 40% of all students attempting the question violated this principle.

Great difficulty was undoubtedly encountered in putting the given figure on paper correctly. The naming of the parts of the figure in the question made up some of this. Such a difficulty can only be overcome by practice.

Within the question itself there is again the difficult conception of the relationship existing between parts and the whole -- in this case, triangles and their parts. This has arisen in several questions and presents a factor that can, and should, be removed. Again, much practice in the various manifestations of this principle is the solution.

Value

3 17. A point D keeps within the $\triangle$ABC and moves so that its distances from AB and AC are always equal. Draw and describe the locus of the point D.



17. The concepts in connection with loci are very limited in number in Grade IX. There is the concept of the term -- "a path traced by a point moving and fulfilling certain conditions". In addition to this, ability is needed to draw one certain required locus. To complete this question, an explanation of the line is necessary.

Results:	City	Town	Rural	Totals
Correct	11	19	18	48
Incorrect	102	99	90	291
Not attempted	37	32	42	111

12% of the pupils, in addition to those who had it correct, received part marks for drawing the bisector of the angle BAC, without any explanation. Over 10% bisected angles ABC or ACB, or both. 20% drew one or more right bisectors of the sides. Over 4% drew the locus as a circle with D, as the centre, placed apparently without rhyme or reason. Almost 10% dropped a line from A perpendicular to AB.

An examination of the answers to this question indicates that most pupils had some ideas about loci, but had them too badly confused to use them successfully. This weakness could be explained on the basis of lack of practice.

Whether pupils know a topic thoroughly or not, they seem to be willing to attempt to answer a question. The large percentage of pupils who did not attempt this question might show that this is one topic that is treated very casually in Grade IX.

Value
10 18. Prove that triangles on the same base and between the same parallels are equal in area.

18. This question could be answered by a re-duplication of a theorem, previously memorized. It is one of the few questions which could be done on the basis of work memorized but not necessarily understood.

It demands a knowledge of the method of stating a formal proof. Clear concepts of the written instructions, of the figure, and of what is given and what is required, are necessary. There must be some construction made. In the section devoted to proof, the theorems that parallelograms on the same base and between the same parallels are equal in area, and, that a diagonal bisects a parallelogram, are required. To arrive at the final conclusion, knowledge of the axioms and assumptions is also necessary.

Results:	City	Town	Rural	Totals
Correct				
(full value)	9	13	19	41
(part marks)	45	54	47	146
Incorrect	82	74	76	232
Not attempted	14	9	8	31

In the group tabulated as "Part marks" fell those answers which contained some minor error, as well as those whose only correct feature was to state the "Given" and "Required" parts.

The fact that over 50% of the students couldn't get any marks on this question proves that they just hadn't done the steady, hard, uninspiring work necessary to learn something obviously fundamental in Geometry. It must not be assumed from such slender evidence alone, that they didn't know the conclusion of this theorem, but it may be assumed, that any or all of the elements used in proving the theorem were, at least, only imperfectly known.

In addition to the question as to whether the elements were known, there is the question as to how many pupils knew the sequence of reasoning necessary to carry the work through to a successful conclusion. 3% knew enough to draw one parallelogram -- but tried to arrive at a conclusion on the basis of one parallelogram and one only. Then again, the concepts of "equal in area" and "congruent" were badly confused. Almost 9% tried to prove the triangles congruent. A very few of these assumed enough in drawing their figure to make it valid, but the great majority just applied the concept of congruency wrongly, on the basis of an unwarranted assumption.

CHAPTER V

STATISTICAL TREATMENT OF SCORES

It was thought fitting that any data derived from a statistical treatment of the Geometry, Algebra, and General Test Scores should be of such a nature that they would be of value to a teacher who is not a specialist in Statistics. Further, along this line, two things may be assumed: firstly, that the teaching done in these subjects is comparable from year to year; secondly, that the pupils who receive the instruction have the same average ability. However, one variable factor does enter in, that is, the degree of difficulty of the examination. Since this variable factor is of great consequence in determining what scores pupils will get on an examination paper, it was not thought fit to treat the scores intensively. Results obtained from intensive treatment of the scores would be of limited value in forecasting future trends, and other uses to which statistical detail can be put.

As a consequence, some comparatively simple operations were carried out: the means, standard deviations and medians of all the samples were found; the rank correlation coefficients were calculated for each sub-group within the sample; the city General Test Scores and city Algebra scores were correlated; and the Algebra and Geometry scores were treated graphically.

The Algebra and Geometry scores were taken from the examination answer papers, and the scores on the General Tests written by the same students were obtained from the Department of Education. In the case of the Algebra paper, the total value of the paper was 83 marks. The Geometry paper had a total value of 79 marks.

TABLE I

MEANS, STANDARD DEVIATIONS, AND MEDIANS OF ALGEBRA AND GEOMETRY SCORES

Nature of the Sample	Arithmetic Mean	Standard Deviation	Median
All the Algebra Scores	35.60#	15.16	35.18
City Algebra Scores	35.00	15.43	34.57
Town Algebra Scores	36.30	14.17	36.94
Rural Algebra Scores	35.45	15.59	34.25
All the Geometry Scores	29.85	14.71	28.59
City Geometry Scores	29.20	14.16	27.50
Town Geometry Scores	31.05	15.04	29.75
Rural Geometry Scores	29.30	14.64	28.75

#(All numbers represent scores.)

Regarding the statistical data derived from the Algebra and Geometry scores, there was very little difference noted between the whole sample constants and sub-group constants. This would indicate that the results of the teaching and the learning in the different school environments, that is, cities, towns, and rural areas, are quite comparable. The city teachers have one advantage, in that they are teaching a limited number of subjects as compared with the town or rural teachers. To counteract this advantage, the latter have one which nearly balances it -- they have limited numbers of students, and those students are more accustomed to carrying on their work more independently of teacher guidance and control. Considerations of advantage or disadvantage in one condition or another are rendered less meaningful by the individual initiative and ability of the students themselves.

TABLE II

RANK CORRELATION COEFFICIENTS OF ALGEBRA AND GEOMETRY SCORES

Nature of Sample	Correlation Coefficient
City Scores	.73
Town Scores	.74
Rural Scores	.72

In each case examined, there appears to be a reasonably high degree of correlation between the scores obtained in Algebra and those obtained in Geometry. This is as one would expect -- similar skills and concepts are used in each subject. Also there seems to be no significant difference in the degree of correlation in the scores of the various sub-groups examined.

The city General Test Scores were correlated with the city Algebra scores. The correlation coefficient was 0.55. This is a moderately high degree of positive correlation, and again indicates what experience would lead one to expect.

A consideration of the following Histograms would indicate that the Algebra paper was the better examination in that the scores were more nearly normally distributed. The Geometry distribution is definitely less normal, indicating that students were not able to achieve the same average results on this paper as on the Algebra.

FIGURE 1

REPRESENTATION OF ALGEBRA SCORES

450 Scores - 1940 Examination

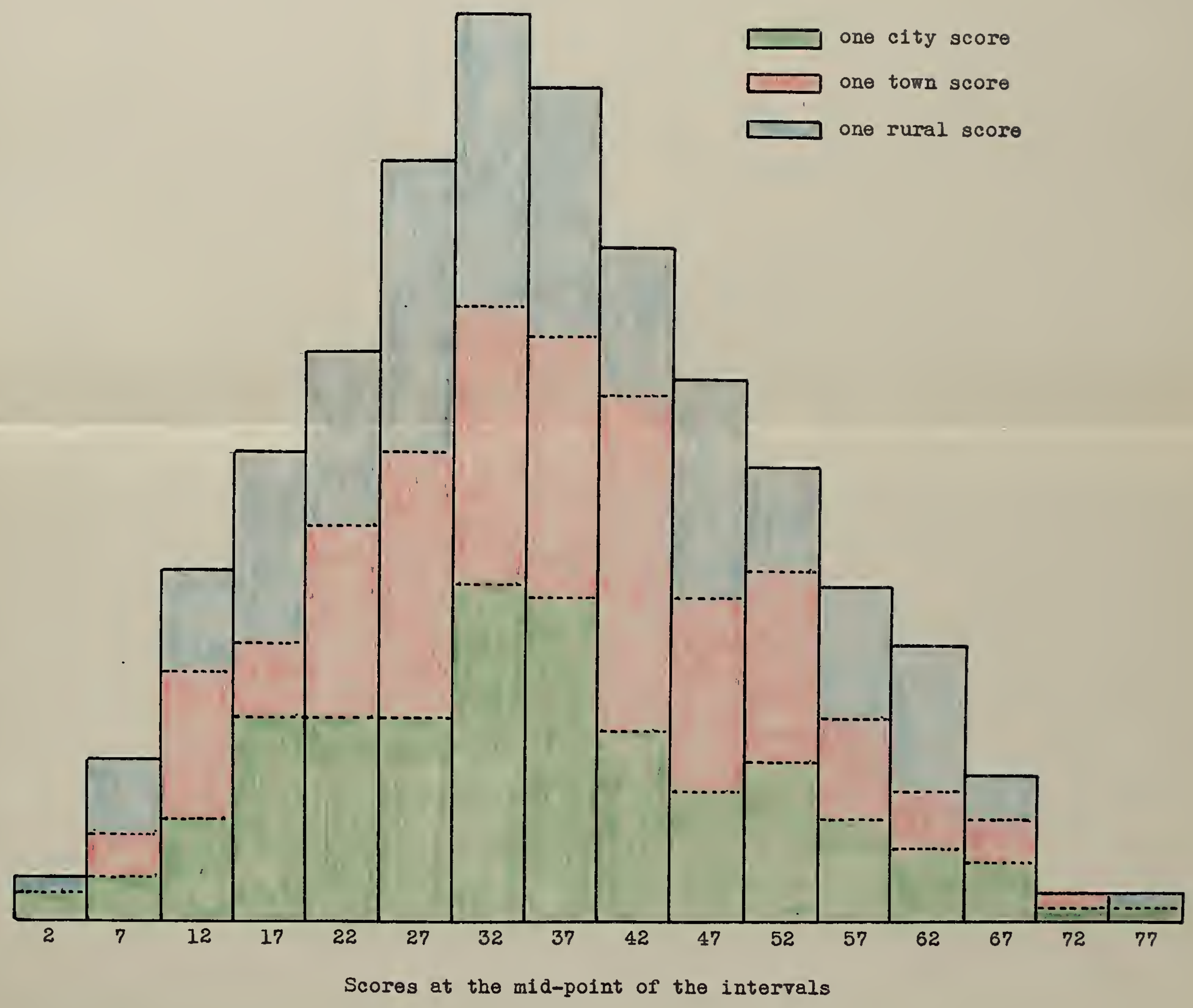
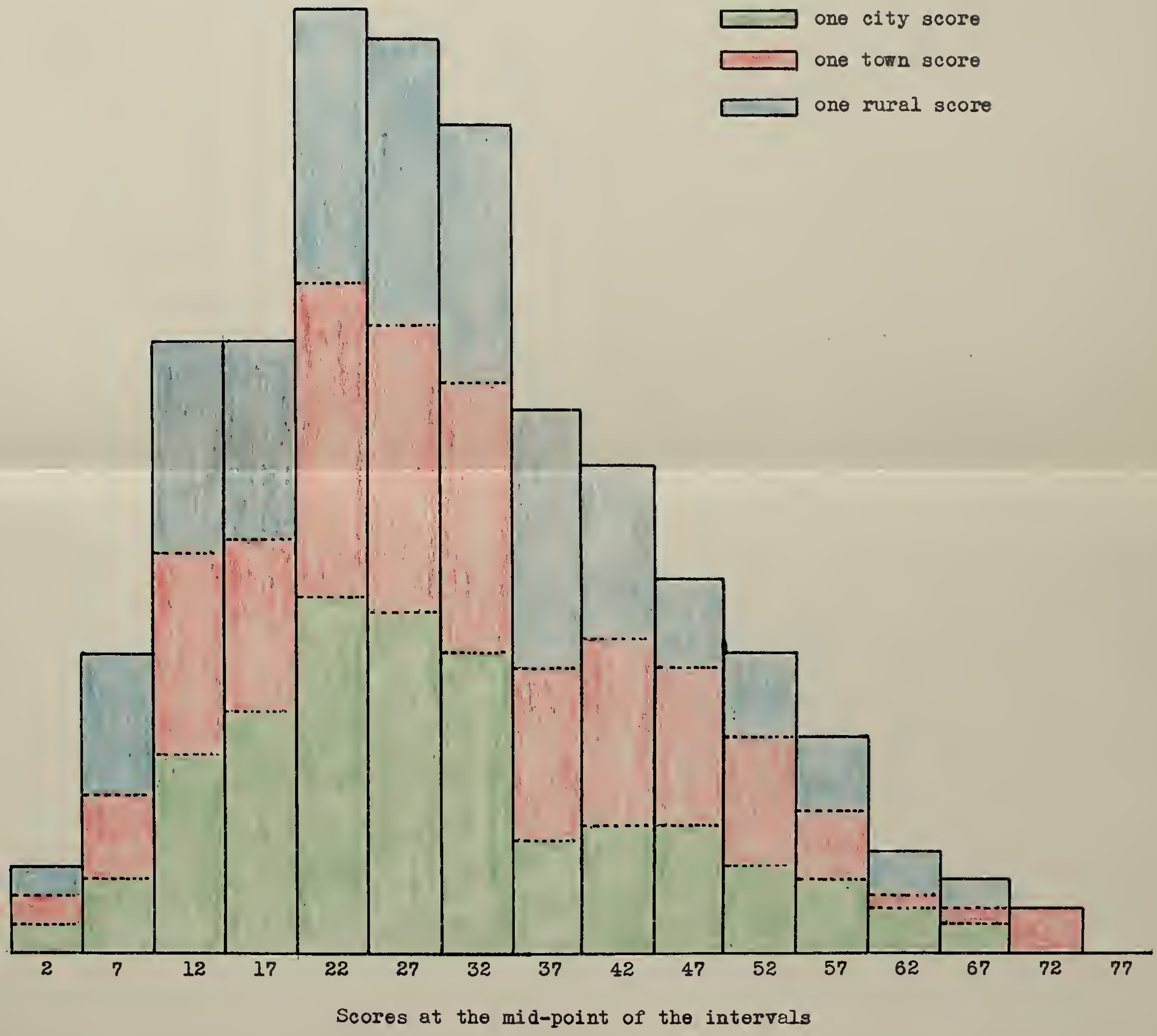


FIGURE 2

REPRESENTATION OF GEOMETRY SCORES

450 Scores - 1940 Examination



CHAPTER VI

SUMMARY, CONCLUSIONS, AND CRITICISMS

General Criticisms of Students' Work, and Suggested Remedial Work

Any person who attempts to carry out an investigation expects to benefit from doing the work. He expects his knowledge to be increased, and his vision to be extended. Such is the case with regard to this investigation. Many ideas formerly held were verified, new concepts came into being, and new problems, so far unanswered, became apparent. Specifically, with regard to Grade IX Mathematics, a clearer understanding of the problems involved, from the teacher's as well as from the student's viewpoint, was gained. An attempt will now be made to summarize some of these findings.

Definitions and terms are seen to require greater emphasis. Undoubtedly many students can repeat a formal definition but cannot put it into practice. Definitions, to become meaningful, must always be used in connection with a situation -- never as an abstraction. Further, they should be applied to situations in which extraneous data are given, in order to give practice in exercising judgment. The ability to choose a fundamental factor from a group of irrelevant details will prove of great value in life situations.

Pupils should be led to realize that, when confronted by a problem, they have numerous and varied methods which can be used in trying to arrive at an answer. Among these methods or devices for gaining help is one that all mature students use to good advantage -- a figure is drawn whenever and wherever possible. If this is done the problem is presented pictorially as well as by means of the printed word. This gives added power, both in the interpretation and in the solution of the

problem. The answer papers gave evidence that this was not done. The pupils had not been made to realize the advantages accruing from a carefully drawn figure.

Another device that should have a much wider application -- one might say it should have a universal application -- is the method of verifying an answer by another procedure. This, in some cases, could be accomplished by making an approximation of the answer. Other problems lend themselves to an entirely new and different approach than the one originally used. It is only with very wide experience that a student builds up a great variety of alternate methods, but Grade IX students should have knowledge of the principle, and be prepared to exercise it within the, perhaps, limited extent of their knowledge of optional methods. It is felt that this is not being done.

Nothing that has been said does away with the need for the pupil to exercise critical judgment. He must be prepared to examine a figure or a problem to determine approximate values. The exercise of this power will ensure that impossible or wholly unreasonable answers will not be countenanced.

Mechanical errors occurred far too frequently on the answer papers. These errors were made in very simple computations, and in more complex ones. The work in connection with a new medium is difficult enough, without the new issues and concepts being clouded by difficulties carried over from previous grades. One way of dealing with this problem would be to test all students entering Grade IX in September in fundamental operations with whole numbers, vulgar and decimal fractions, and percentage fractions. Those who fell below a reasonably high standard, say seventy-five per cent accuracy, should be obliged to

do extra work in these fields, in order that their attainment be brought up to a reasonable standard. This imposes a further task on the already over-worked teacher, but the increased efficiency might compensate at a later date for the extra work. It is firmly believed that mistakes in calculation can be and should be eliminated -- and this at the earliest possible time.

When there are several ways of doing one thing, it is believed that the one with the most universal application should be stressed, and the others used as auxiliary methods. Naming angles is an example of this principle. Three or four methods are used, but the one requiring three letters is the most comprehensive. If the three or four methods are used casually, without any emphasis being given to the most important one, some pupils will not be able to proceed when a situation arises in which the three letter method must be used. It is believed that this principle has quite a wide application in Algebra as well as in Geometry.

The whole question of English is very closely related to Mathematics. The two aspects are: expression of a concept, and interpretation of the printed word. More emphasis should be placed on these phases of the work. No doubt all teachers realize this, and all are convinced of the need to correlate English with Mathematics. But are all teachers convinced of the need to correlate Mathematics with English? Do all teachers use, in suitable proportion, mathematical material in the English class? In a school system where the work is departmentalized the Mathematics teacher is cautioned to stress English, but the English teacher is not generally under any obligation to give exercises in English, related to Mathematics. The more varied the approaches,

[illegible]

[illegible]

[illegible]

the greater the impact of a principle on a student.

The concepts embodied in the relationship of the whole to its parts presented difficulty to a large number of students. In Geometry Question 3, the area of two shaded parts was to be found by subtracting unshaded parts from the whole. In Question 7, the area of a ring had to be found by subtracting one circle from a larger one. Again, in many geometric figures of a more technical nature, these same concepts were needed in the solution of the problem. Students have not mastered these relationships. Some examples in the authorized text illustrate this principle, but there are not enough of them to establish it in the minds of students. Sufficient drill should be given to ensure that pupils are aware of it before leaving Grade IX as it has a wide application in later mathematics and in life situations.

Prior to Grade IX, most of the Geometry used by our students was concerned with measuring lines and angles. In Grade IX, the complementary method, that of calculating sizes of lines, angles and areas is introduced. The former method is not to be discarded with the advent of the new one. Both are to be retained but the one to be used will be determined by the particular situation. The answers examined show that the students have not acquired the ability to differentiate between the situations in which each method is appropriate -- they measure when they should calculate, and calculate when they should measure.

One aspect of Geometry is its use as a logical system of reasoning. The first requirement in this system, is that the material given must be clearly understood. All the implications and ramifications of an hypothesis must be known if logical reasoning is to be developed from it. One error in reasoning that students frequently make, is the de-

[illegible]

[illegible]

[illegible]

riving of a conclusion from an unwarranted assumption. Proper understanding of an hypothesis would obviate most errors of this nature. Further, many geometric proofs are the result of a chain of reasoning -- each additional statement depends upon the preceding statements with their reasons. This work is difficult for Grade IX students. They haven't grasped the material or the principle of Geometry, as a system of reasoning as well as they should. Very intensive and extensive drill in all branches of reasoning should be given.

One aspect of this question is still being debated in academic circles. One school of thought insists that Geometry is a study designed to train for logical thinking. The other school of thought believes that logical thinking is a training for Geometry, and that only those students who intend using Geometry in a profession should study it. Be that as it may, we in Alberta are desirous of using this subject in Grade IX to the greatest degree possible. All students are expected to acquire certain geometric facts and the method of reasoning on sound hypotheses. Some will, no doubt, be able to transfer the training received in Geometry to logical thinking in other spheres, because of the effort of the teachers in treating the subject, to relate the two. In either event much practice is needed to achieve good results.

From examination of the results of all questions, one is forcibly struck by the large proportion of students who attempt each question, and the small proportion who are able to arrive at the correct answer. No doubt, some teachers encourage their students to try any and all questions. Disregarding this factor, the situation still remains very challenging. In a great many cases, the student not only attempts the question but proceeds to carry his work through to an answer. Often

the method as well as the answer is incorrect, but the student is apparently unaware of either mistake. When this student reads the question he is apparently under the impression that he knows how to do it. He may be in possession of a very hazy outline of how the question should be worked, or have an equally hazy notion of a method which he thinks is applicable. In either case, he proceeds to use his method and arrives at an answer which is utterly and completely wrong. But he does not realize that he is wrong and submits the answer. It is with this person "who is wrong but knows not that he is wrong" with whom we are concerned at the moment. Possibly, quite a large percentage of pupils try an examination with a false impression of their ability in a subject. It is believed that this misplaced self-confidence resulted in the attempt being made which gave the impossible answer, and it also was the cause of the student's failure to master the concepts presented to him throughout the year. Probably, he saw the point superficially at the time it was presented, thought it was easy, and neglected to do enough work with the concept to master it. The teacher also might be partially responsible for this misplaced confidence. If the teacher demonstrates too freely instead of putting the onus on the student, he is robbing the student of an opportunity to gain real power. It was Henry Ford who said, "The only thing you can give a man without doing him an injustice is an opportunity".

One aim of all education is to inculcate in the student desirable habits and attitudes. In Mathematics one very desirable habit is that students put work down neatly. It should be concise -- in so far as the very usefulness of Mathematics is its use of symbols -- but only to the extent that nothing essential is omitted. Many students show

that by the end of Grade IX they have acquired these desirable habits and skills. Others lose credits by their very lack of neatness -- they do not receive the benefits from work planned well, and neatly executed. Eternal vigilance, on the part of the teachers is needed to develop these habits in the pupils.

Teachers must plan their work in Grade IX Mathematics very carefully, in view of the limited time at their disposal. Insufficient and improper drill are causes of much of the poor learning that is in evidence on examination papers. To make the work more meaningful and to ensure that concepts will be retained, oral exercises dealing with simple quantities will accomplish the greatest good for any given amount of time spent. Concepts must eventually be raised to the mental level if they are to be of real value, and oral work is a prime means to this end.

Some Suggestions in Regard to Marking of Examination Papers

Throughout many long years of setting and marking papers, our Department of Education officials have evolved a system of marking that is reasonably adequate. Many precautions are taken to ensure that the candidates get full credit for their work.

The investigator has never marked Departmental examination papers, so anything that might be said in this connection comes as a result of experience with the papers that have been used in this study. However, it is impossible to leave the investigation without making some observations about this part of the work. It is believed that these observations affect the Grade IX students in the Province of Alberta.

Values of questions are sometimes changed by the marking committees after the papers have been written. One purpose served by

putting values beside questions is to indicate to the student something of the relative importance or merit of each question. If it is subsequently changed, previous calculations as to time allotments, etc., are rendered less valid. This is a very minor point, but it seems as if the proper and final value should be placed on a question at the start -- so much care does go into the formulation of these papers.

There are errors on the part of the markers in awarding the proper value to an answer. It is understandable how such errors occur -- the work is done by teachers who have just completed ten months of exacting work in the, generally, very warm month of July, and under pressure. This human equation will always be present. The Department makes tacit recognition of this by providing checkers -- to check the examiners' addition of the marks awarded. There are surprisingly few errors in addition, but there are too many errors in the evaluation of answers. No attempt was made to record the errors noted. However, it is not the fact that there is any particular percentage of errors in marking, but the fact that there are any errors. It can be presumed, for lack of objective evidence to the contrary, that these errors work to the detriment of students at least as often as they work to his advantage.

An examiner is expected to mark in the case of the Algebra, forty-two separate answers and in the case of Geometry, thirty-four separate answers. In addition, he must be prepared to recognize alternate answers, and to award partial marks for work carried correctly at least part of the way. This seems to be the cause of errors.

To overcome this defect, an alternate method of marking is herewith submitted. One examiner might be assigned to one question, or one part of a question where several independent answers are given. In a

comparatively short time he could analyse that question, determine the nature of the mistakes, and, in consultation with the chairman determine a just distribution of partial marks and alternate forms in which the correct answer might occur. Then, when all examiners had been so prepared, one examiner would mark the question which he had studied, and that question only. He would be called upon to remember a very limited number of acceptable answers and their appropriate values. If a certain question took much more time to mark than another, a battery of markers might be assigned to work on one question. All the aspects of this system of marking have not been thought out, for example, how the papers could be kept in numerical order in passing a battery of markers. This method should be able to reduce the number of errors in marking to an absolutely irreducible minimum.

Some Observations Regarding Examinations

The short answer type of examination has had a very rapid rise in popularity in recent years. This type has many advantages that have been stressed at great length by many writers. It tests ability to think rather than ability to memorize a given body of data. It puts a premium on good reading and comprehension of subject matter. In Mathematics, it allows for expression of mathematical concepts in symbols, and thus is an advantage to that pupil who is deficient in expressive English. The marking of the answer papers is put on a more objective basis than formerly.

There are disadvantages, also given prominence by some writers. This type of test is so concerned with testing ability to think, that it makes insufficient provision for testing ability to make sustained effort

in any one sphere of work. It must be noted that some Algebra and Geometry questions purport to test this very thing -- but the examination papers show that many students do not attempt these long questions, or else start them and abandon them after a futile effort. This seems a very definite weakness, in that much subsequent work in Mathematics is dependent on ability to carry a long complicated task through several phases correctly. How do the examiners know, in evaluating a pupil's work solely on the basis of an objective test that a pupil leaving Grade IX is able to do a piece of work requiring this very necessary ability?

Everyone has often heard the criticism that under a new programme of studies, or a new system of examinations, pupils are not doing as well as they did under the former systems. One is reminded of a story that concerns the editor of Punch. Someone taxed him that "Punch is not as funny as it used to be" to which the editor is supposed to have remarked: "No, it never was".

Before criticizing adversely our present methods of instruction and examination, many factors should be considered. Formerly, a selected body of students formed the population in Grade IX -- they met an examination at the end of the Grade VIII level that caused the weakest to repeat the grade or drop out of school. Now, almost all our school population remains in school to the end of the Grade IX level, and the people who were formerly eliminated prior to this stage, now form a part of those writing the examination. Twenty years ago, much more emphasis was placed on the fundamentals -- English, Mathematics, Science, Languages and the Social Studies. Now, there are many more subjects and interests with a corresponding shortening of the time that can be placed on any of the subjects. Much more emphasis is placed upon training in

citizenship, in personality, and in wise use of leisure time. Lastly, there are students in Grade IX now, who are doing their work as neatly and as conscientiously, and receiving marks on examinations equally as high as any attained in former generations of school populations.

The officials and staff of the College of Education, University of Alberta, have contributed, in a large measure, to any success that may have been attained by this report. Throughout the preparations for, and the carrying on of the work, help and constructive criticism have always been freely granted. One can but say "Thank you" to those people whose assistance and inspiration has been so freely given.

The officials of the Department of Education have been very kind in affording access to the papers used in carrying on this study. The thanks of the investigator is hereby tended to them for their whole-hearted co-operation.

APPENDIX

EXAMINATION PAPER NUMBERS AND SCORES OF PUPILS

City Students' Results

Candidate's Number	Algebra Score	Geometry Score	General Test Score
511	17	13	17
531	50	47	52
551	30	12	88
571	28	26	96
591	50	20	62
611	38	47	81
631	60	46	78
651	23	27	89
671	21	12	62
691	43	32	62
711	36	22	62
731	16	14	48
751	31	26	37
771	41	25	48
791	61	44	83
811	46	57	30
831	39	57	92
851	29	18	41
871	25	23	55
891	38	30	57
911	15	29	62
931	35	23	12
951	32	32	55

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Candidate's Number	Algebra Score	Geometry Score	General Test Score
971	21	22	41
991	15	12	17
1011	22	19	48
1031	49	46	89
1051	59	61	57
1071	52	20	55
1091	59	20	78
1111	60	41	99
1131	51	42	92
1151	44	30	85
1171	53	27	74
1191	32	30	55
1211	31	4	65
1231	6	8	3
1251	56	54	99
1271	38	29	62
1291	53	57	99
1311	34	26	50
1331	31	31	44
1351	39	30	48
1371	45	42	88
1391	12	18	22
1411	20	31	3
1431	38	19	22
1451	3	12	5
1471	34	23	44

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Candidate's Number	Algebra Score	Geometry Score	General Test Score
1491	6	23	30
1511	31	15	30
1531	36	28	44
1551	67	47	99
1571	37	31	71
1591	68	51	98
1611	18	15	15
1631	40	28	44
2319	17	21	57
2339	55	38	57
2359	43	15	25
2379	37	35	44
2399	24	16	62
2418	15	9	25
2419	30	17	65
2438	23	12	25
2458	35	28	52
2478	18	22	9
2498	58	53	83
2518	34	22	57
2538	31	29	37
2558	45	55	74
2578	31	21	17
2598	62	66	68
2618	44	40	85
2638	32	19	78

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Candidate's Number	Algebra Score	Geometry Score	General Test Score
2658	42	27	74
2678	24	13	27
2698	49	41	92
2718	43	59	33
2738	38	39	81
2758	47	48	68
2778	21	30	81
2798	40	34	33
2818	41	38	20
2838	15	9	48
2858	31	30	81
2878	31	31	71
2898	16	20	74
2918	44	30	65
2938	31	13	37
2958	38	19	15
2978	18	19	20
2998	13	33	78
3018	27	13	15
3038	30	25	9
3058	20	8	5
3078	14	13	12
3098	67	61	71
3118	16	8	22
3138	41	46	74
3158	33	47	37

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Candidate's Number	Algebra Score	Geometry Score	General Test Score
3178	68	61	98
3198	23	23	9
3218	39	25	71
3238	29	19	78
3258	29	21	44
3278	11	12	12
3298	24	22	57
3318	21	28	85
3338	35	29	52
3358	38	34	92
3378	50	32	62
3398	48	31	55
3418	77	66	99
3438	50	38	55
3458	72	54	100
3478	11	11	68
3498	29	18	15
3518	27	31	81
3538	52	36	94
3558	29	30	41
3578	18	19	12
3598	36	41	85
3618	4	4	1
3638	64	44	41
3658	9	26	25
3678	54	39	97

Candidate's Number	Algebra Score	Geometry Score	General Test Score
3698	37	47	74
3718	58	51	81
3738	23	27	41
3756	53	44	37
4489	26	29	48
4509	14	17	8
4529	29	16	62
4549	37	23	44
4569	34	32	81
4589	11	22	17
4609	37	20	17
4629	48	25	81
4649	43	39	12
4669	15	11	41
4689	26	22	44
4846	31	24	71
4866	33	20	68
4886	28	26	25
4906	46	26	62
4926	27	23	44
4946	39	50	30
4966	56	28	52
4986	30	22	71

Town Students' Results

Candidate's Number	Algebra Score	Geometry Score	General Test Score
1	38	37	78
20	35	27	25
40	23	27	15
60	44	31	65
80	30	28	74
100	37	35	41
120	33	29	17
140	33	25	17
159	56	34	57
180	52	57	99
200	51	52	83
220	33	29	78
240	63	54	81
260	37	23	20
280	25	13	12
300	34	10	20
320	43	35	68
340	44	26	81
440	37	40	97
460	44	22	48
480	43	35	3
500	5	4	17
1637	41	40	88
1657	13	13	15
1677	25	12	68

Candidate's Number	Algebra Score	Geometry Score	General Test Score
1697	20	19	30
1717	53	24	41
1737	38	27	22
1757	50	34	41
1777	26	28	15
1797	40	32	17
1816	21	17	9
1836	53	36	81
1856	45	48	65
1876	28	16	22
1896	41	32	33
1916	14	41	12
1936	39	51	41
1956	51	51	81
1976	36	22	15
1996	33	35	81
2016	22	35	30
2036	62	45	89
2056	59	40	89
2076	32	47	44
2096	48	43	81
2116	71	69	83
2136	42	47	74
2156	58	55	92
2176	27	21	41
2196	33	23	65

Candidate's Number	Algebra Score	Geometry Score	General Test Score
2216	21	8	33
2236	21	23	55
2256	40	23	20
2276	40	35	83
2296	62	55	83
2316	31	26	22
3757	31	20	62
3777	24	17	37
3797	46	45	99
3817	29	23	1
3837	14	11	41
3857	19	17	12
3877	45	49	68
3897	35	32	52
3917	27	22	55
3937	41	37	20
3957	36	37	62
3977	34	37	30
3997	31	16	57
4017	42	24	65
4037	28	31	37
4057	14	26	68
4077	32	7	44
4097	53	56	78
4117	14	13	37
4137	67	52	62

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Candidate's Number	Algebra Score	Geometry Score	General Test Score
4157	13	12	12
4177	55	56	68
4197	18	23	57
4216	40	50	2
4236	35	23	37
4256	29	26	30
4276	51	44	65
4296	53	48	83
4316	25	20	85
4336	47	50	68
4356	57	44	17
4376	21	7	85
4396	32	23	25
4416	47	40	52
4436	58	71	98
4456	36	22	37
4476	7	31	20
4706	23	16	3
4726	48	30	37
4746	62	72	100
4766	51	30	62
4786	19	16	37
4806	44	34	55
4826	16	5	25
4845	49	49	12
4995	11	10	3

Candidate's Number	Algebra Score	Geometry Score	General Test Score
5015	39	40	52
5035	29	20	62
5055	44	27	25
5075	40	52	81
5095	20	20	48
5115	42	48	62
5135	37	47	52
5155	37	27	71
5175	51	33	81
5195	28	37	25
5215	31	23	12
5235	47	45	71
5255	66	64	89
5275	33	25	94
5295	32	29	8
5315	26	11	30
5335	29	32	48
5355	13	10	17
5375	26	25	88
5395	42	20	30
5415	41	9	8
5435	35	41	74
5455	51	33	30
5475	8	13	4
5495	69	70	88
5515	45	31	20

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Candidate's Number	Algebra Score	Geometry Score	General Test Score
5535	29	19	17
5555	36	18	12
5576	33	17	6
5596	49	50	55
5616	14	34	96
5636	46	34	57
5656	13	8	1
5676	58	43	62
5696	41	25	9
5716	51	44	71
5736	32	11	41
5756	43	31	62
5776	35	27	30
5796	22	17	20
5816	22	10	6
5836	41	43	57
5856	29	4	12
5876	29	27	92
5896	17	10	3
5916	23	24	30
5936	49	34	88

Rural Students' Results

Candidate's Number	Algebra Score	Geometry Score	General Test Score
6661	3	5	5
6681	27	25	15

Candidate's Number	Algebra Score	Geometry Score	General Test Score
6701	62	60	88
6721	67	56	97
6741	33	18	12
6761	14	18	25
6781	27	26	3
6801	52	51	62
6821	20	25	62
6841	21	13	15
6861	54	18	74
6881	55	55	74
6901	51	38	44
6921	26	22	12
6941	61	36	33
6961	59	35	37
6981	49	50	71
7001	15	8	8
7021	60	30	71
7041	38	21	30
7061	42	51	55
7081	62	69	96
7101	16	29	31
7121	34	30	33
7141	31	20	5
7161	61	31	96
7181	65	52	92
7201	36	29	41

Candidate's Number	Algebra Score	Geometry Score	General Test Score
7221	21	29	41
7241	23	28	33
7261	34	40	15
7281	30	24	44
7301	30	25	25
7321	26	34	20
7341	26	22	62
7361	58	67	55
7381	7	12	2
7401	48	43	25
7421	32	9	12
7441	45	47	27
7461	29	25	9
7481	16	12	44
7501	49	27	74
7521	9	11	44
7541	16	23	17
7561	7	13	17
7581	35	28	74
7601	46	32	52
7621	15	17	6
7641	25	36	85
7661	15	31	5
7681	30	33	74
7701	35	18	44
7721	32	35	48

Candidate's Number	Algebra Score	Geometry Score	General Test Score
7741	28	29	12
7761	43	35	41
7781	65	60	65
7801	60	44	48
7821	7	13	5
7841	36	37	65
7861	38	35	20
7881	31	15	6
7901	32	46	30
7921	37	26	25
7941	55	55	85
7961	17	24	4
7981	57	42	55
8001	17	9	33
8021	21	13	15
8041	51	44	85
8061	45	34	88
8081	58	36	85
8101	58	56	52
8121	34	24	74
8141	34	36	25
8161	28	40	37
8181	28	31	57
8201	35	18	65
8221	33	17	12
8241	25	8	37

Candidate's Number	Algebra Score	Geometry Score	General Test Score
8261	42	7	41
8281	20	29	9
8301	62	60	92
8321	13	11	9
8341	44	37	15
8361	31	38	48
8381	28	43	55
8401	32	30	37
8421	47	38	85
8441	39	22	62
8461	42	17	85
8481	36	33	12
8501	19	9	1
8521	23	7	8
8541	36	23	30
8561	13	3	1
8581	56	41	81
8601	46	39	89
8621	54	29	3
8641	16	29	37
8661	47	21	41
8681	38	26	3
8701	26	18	83
8721	39	45	81
8741	19	24	33
8761	41	22	81

Candidate's Number	Algebra Score	Geometry Score	General Test Score
8781	20	4	12
8800	20	12	20
8801	12	13	27
8820	44	16	33
8840	28	20	8
8860	15	19	9
8880	8	11	9
8900	60	46	100
8920	45	42	55
8940	42	41	27
8960	28	22	20
8980	27	27	20
9000	26	36	48
9020	34	33	62
9040	28	23	37
9060	23	24	48
9080	21	12	20
9100	41	36	44
9120	59	59	62
9140	14	5	1
9160	53	25	2
9180	30	21	20
9200	61	46	74
9220	13	9	9
9240	45	17	30
9260	36	31	33

Candidate's Number	Algebra Score	Geometry Score	General Test Score
9280	42	35	71
9300	34	24	37
9320	30	54	15
9340	60	47	92
9360	35	33	62
9380	47	39	57
9400	51	43	92
9420	19	18	12
9440	75	54	62
9460	46	34	48
9480	39	39	52
9500	24	25	22
9520	27	13	8
9540	38	44	57
9560	25	11	48
9580	45	31	74
9600	48	30	17
9620	13	14	30

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